	E.G.S. PILLAY ENGINEERING COLLEGE (An Autonomous Institution, Affiliated to Anna University, Chennai) Nagore Post, Nagapattinam – 611 002, Tamilnadu.	Rev.0 COE/2017/QB

1701CA101 MATHEMATICAL FOUNDATIONS OF COMPUTER APPLICATIONS				
Academic Year :	**2018-2019**	**Question Bank**	**Programme :**	**MCA**
Year / Semester :	**I / I**		**Course Coordinator:**	

Course Objectives	Course Outcomes
The primary objectives of this course to provide mathematical background and sufficient experience on various topics of discrete mathematics like matrix algebra, logic and proofs, combinatorics, graphs, algebraic structures, formal languages and finite state automata. This course will extend student's logical and mathematical maturity and ability to deal with abstracting and to introduce most of the basic terminologies used in computer science courses and application of ideas to solve practical problems.	After completing this course, students should demonstrate competency in the following skills: CO 1: Apply the knowledge of matrix, functions and relations concepts needed for designing and solving problems (K3) CO 2: Relate logical operations and predicate calculus needed for computing skill (K2) CO 3: Interpret the validity of verbal or symbolic arguments using rules of inference (K2) CO 4: Construct and solve Boolean functions for defined problems (K3) CO 5: Comprehend the algebraic structure with their applications to handle algebraic spaces (K2) CO 6: Apply the acquired knowledge of finite automata theory and to design discrete problems to solve by computers (K3)

PART – A (2 Mark Questions With Key)

S.No	Questions	Mark	Cos	BTL
UNIT I – MATRIX ALGEBRA				
1	Define Diagonal matrix with example	2	1	K1
	In a square matrix all the elements except elements in the main diagonal are zeros, then the matrix is called a diagonal matrix	1		
	$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{pmatrix}$	1		
2	Define Symmetirc matrix with example	2	1	K1
	A matrix is symmetric, if $A = A^T$	1		

	$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{pmatrix};\ A^T = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{pmatrix}$	1		
3	Define Skew Symmetirc matrix with example	2	1	K1
	A matrix is symmetric, if A = - A^T	1		
	$A = \begin{pmatrix} 0 & 3 & 2 \\ -3 & 0 & 5 \\ -2 & -5 & 0 \end{pmatrix};\ -A^T = \begin{pmatrix} 0 & 3 & 2 \\ -3 & 0 & 5 \\ -2 & -5 & 0 \end{pmatrix}$	1		
4	Define singular matrix and nonsingular matrix	2	1	K1
	A square matrix is said to be singular if $\lvert A \rvert = 0$	1		
	A square matrix is said to be non-singular if $\lvert A \rvert \neq 0$	1		
5	Define Hermitian matrix and Skew Hermitian matrix	2	1	K1
	A square matrix is said to be Hermitian if $A = (\bar{A})^T$ A square matrix is said to be Skew Hermitian if $(\bar{A})^T = -A$	1 1		
6	Define Unitary matrix	2	1	K1
	A square matrix A is said to be unitary if $A\,(\bar{A})^T = I$	1		
7	Check whether the matrix B is orthogonal? $\boldsymbol{B} = \begin{pmatrix} \boldsymbol{cos\theta} & \boldsymbol{sin\theta} & \mathbf{0} \\ \boldsymbol{-sin\theta} & \boldsymbol{cos\theta} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{pmatrix}$	2	1	
	Condition of orthogonal matrix is $AA^T = A^TA = I$ Here, $BB^T = B^TB = I$ $B = \begin{pmatrix} cos\theta & sin\theta & 0 \\ -sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix};\ B^T = \begin{pmatrix} cos\theta & -sin\theta & 0 \\ sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $BB^T = \begin{pmatrix} cos^2\theta + sin^2\theta & 0 & 0 \\ 0 & sin^2\theta + cos^2\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I$ Similarly, $B^TB = I$. Therefore, the given matrix is orthogonal	1		
8	Define the rank of a matrix	2	1	K1
	The order of highest non-zero minor is known as rank of a matrix	1		
9	Find the rank of $\boldsymbol{A} = \begin{pmatrix} \mathbf{2} & \mathbf{-1} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} & \mathbf{2} \end{pmatrix}$	2	1	K2

	A is of type 2 x 3 , take any 2 x 2 matrix $\begin{vmatrix}-1 & 1\\ 0 & 2\end{vmatrix} = -2 \neq 0$ ⇨ Rank of A = 2	1		
10	Find the rank of $\boldsymbol{A} = \begin{pmatrix}\mathbf{1} & \mathbf{3} & \mathbf{0}\ \mathbf{5}\\ \mathbf{2} & \mathbf{-1} & \mathbf{4}\ \mathbf{2}\end{pmatrix}$	2	1	
	A is of type 2 x 4 , take any 2 x 2 matrix $\begin{vmatrix}0 & 5\\ 4 & 2\end{vmatrix} = -20 \neq 0$ ⇨ Rank of A = 2	1		K1
11	Find K so that the rank of the matrix $\begin{pmatrix}\mathbf{2} & \mathbf{1} & \mathbf{-1}\\ \mathbf{1} & \mathbf{4} & \mathbf{2}\\ \mathbf{3} & \mathbf{5} & \boldsymbol{K}\end{pmatrix}$ is 2.	2	1	
	$\begin{vmatrix}2 & 1 & -1\\ 1 & 4 & 2\\ 3 & 5 & K\end{vmatrix} = 0$ ⇔ 2(4K-10)-(K-6)-(5-12) = 0; ⇔ 8K-20-K+6+7 ⇔ = 0 7K-7 ⇔ =0 K = 1	1 1		K2
12	Find the rank of the matrix A = $\begin{bmatrix}\mathbf{1} & \mathbf{2} & \mathbf{3}\\ \mathbf{1} & \mathbf{4} & \mathbf{2}\\ \mathbf{2} & \mathbf{6} & \mathbf{5}\end{bmatrix}$	2	1	
	$A \sim \begin{bmatrix}1 & 2 & 3\\ 0 & 2 & -1\\ 0 & 2 & -1\end{bmatrix}$ $R_2 \to R_2 - R_1$; $R_3 \to R_3 - 2R_1$ $\sim \begin{bmatrix}1 & 2 & 3\\ 0 & 2 & -1\\ 0 & 0 & 0\end{bmatrix}$ $R_3 \to R_3 - R_2$ The number of non-zero rows is 2; Rank of A = 2	1 1	3	K2
13	State Cayley-Hamilton theorem.	2	1	K2
	Every square matrix satisfies its own characteristic equation	2		
14	Find the characteristic equation of the matrix $\begin{pmatrix}\mathbf{-2} & \mathbf{2} & \mathbf{-3}\\ \mathbf{2} & \mathbf{1} & \mathbf{-6}\\ \mathbf{-1} & \mathbf{-2} & \mathbf{0}\end{pmatrix}$	2	1	
	The characteristic equation is $\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$ Where S_1 = sum of the main diagonal elements = (-2) +1+0 = -1 S_2 = Sum of the minors of the main diagonal elements $= \begin{vmatrix}1 & -6\\ -2 & 0\end{vmatrix} + \begin{vmatrix}-2 & -3\\ -1 & 0\end{vmatrix} + \begin{vmatrix}-2 & 2\\ 2 & 1\end{vmatrix}$	1 1		K1

<table>
<tr><td></td><td>= (0-12) + (0-3) + (-2-4) = -12-3-6=-21
$S_3 = |A| = \begin{vmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{vmatrix} = 45$
Therefore the characteristic equation is $\lambda^3 + \lambda^2 - 21\lambda - 45 = 0$</td><td></td><td></td><td></td></tr>
<tr><td>15</td><td>Find the characteristic equation of the matrix $\begin{pmatrix} \mathbf{1} & \mathbf{2} \\ \mathbf{0} & \mathbf{2} \end{pmatrix}$</td><td>2</td><td rowspan="2">1</td><td rowspan="2">K2</td></tr>
<tr><td></td><td>The characteristic equation of A is $\lambda^2 - S_1\lambda + S_2 = 0$
$S_1 = 3$; $S_2 = 2$
Hence the required characteristic equation is $\lambda^2 - 3\lambda + 2 = 0$</td><td>1
1</td></tr>
<tr><td colspan="5">UNIT II – LOGIC</td></tr>
<tr><td>1</td><td>Write the truth table for conjunction</td><td>2</td><td rowspan="2">2,3</td><td rowspan="2">K1</td></tr>
<tr><td></td><td><table><tr><td>p</td><td>q</td><td>$p \wedge q$</td></tr><tr><td>T</td><td>T</td><td>T</td></tr><tr><td>T</td><td>F</td><td>F</td></tr><tr><td>F</td><td>T</td><td>F</td></tr><tr><td>F</td><td>F</td><td>F</td></tr></table></td><td>2</td></tr>
<tr><td>2</td><td>Write the truth table for Disjunction</td><td>2</td><td rowspan="2">2,3</td><td rowspan="2">K1</td></tr>
<tr><td></td><td><table><tr><td>p</td><td>q</td><td>$p \vee q$</td></tr><tr><td>T</td><td>T</td><td>T</td></tr><tr><td>T</td><td>F</td><td>T</td></tr><tr><td>F</td><td>T</td><td>T</td></tr><tr><td>F</td><td>F</td><td>F</td></tr></table></td><td>2</td></tr>
<tr><td>3</td><td>Write the truth table for conditional statement</td><td>2</td><td rowspan="2">2,3</td><td rowspan="2">K1</td></tr>
<tr><td></td><td><table><tr><td>p</td><td>q</td><td>$p \to q$</td></tr><tr><td>T</td><td>T</td><td>T</td></tr><tr><td>T</td><td>F</td><td>F</td></tr><tr><td>F</td><td>T</td><td>T</td></tr><tr><td>F</td><td>F</td><td>T</td></tr></table></td><td>2</td></tr>
<tr><td>4</td><td>Write the truth table for Bi-conditional statement</td><td>2</td><td rowspan="2">2,3</td><td rowspan="2">K1</td></tr>
<tr><td></td><td><table><tr><td>p</td><td>q</td><td>$p \leftrightarrow q$</td></tr></table></td><td>2</td></tr>
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<tr><td></td><td><table><tr><td>T</td><td>T</td><td>T</td></tr><tr><td>T</td><td>F</td><td>F</td></tr><tr><td>F</td><td>T</td><td>F</td></tr><tr><td>F</td><td>F</td><td>T</td></tr></table></td><td></td><td></td><td></td></tr>
<tr><td rowspan="2">5</td><td>Construct the truth table for $\urcorner \boldsymbol{P} \wedge \urcorner \boldsymbol{Q}$</td><td>2</td><td rowspan="2">2,3</td><td rowspan="2">K2</td></tr>
<tr><td><table><tr><td>p</td><td>q</td><td>$\urcorner P$</td><td>$\urcorner Q$</td><td>$\urcorner P\wedge\urcorner Q$</td></tr><tr><td>T</td><td>T</td><td>F</td><td>F</td><td>F</td></tr><tr><td>T</td><td>F</td><td>F</td><td>T</td><td>F</td></tr><tr><td></td><td>T</td><td>T</td><td>F</td><td>F</td></tr><tr><td>F</td><td>F</td><td>T</td><td>T</td><td>T</td></tr></table></td><td>2</td></tr>
<tr><td rowspan="2">6</td><td>Write the following statement in symbolic form.
If either Jerry takes calculus or Ken takes sociology, then Larry will take English</td><td>2</td><td rowspan="2">2,3</td><td rowspan="2">K1</td></tr>
<tr><td>J: Jerry takes Calculus
K: Ken takes Sociology
L: Larry takes English
$\therefore (J \vee K) \rightarrow L$</td><td>2</td></tr>
<tr><td rowspan="3">7</td><td>Write the following statement in symbolic form.
Mark is neither rich nor happy
Mark is poor or he is both rich and unhappy</td><td>2</td><td rowspan="3">2,3</td><td rowspan="3">K1</td></tr>
<tr><td>R: Mark is rich, H: Mark is happy
$\urcorner R \vee \urcorner H$</td><td>1</td></tr>
<tr><td>$\urcorner R \vee (R \wedge \urcorner H)$</td><td>1</td></tr>
<tr><td rowspan="2">8</td><td>Write the following statement in symbolic form.
Jack and Jill went up the hill</td><td>2</td><td rowspan="2">2,3</td><td rowspan="2">K1</td></tr>
<tr><td>P→ Jack went up the hill
Q → Jill went up the hill
Symbolic form is P∧ Q</td><td>2</td></tr>
<tr><td rowspan="2">9</td><td>State Free and Bound Variables</td><td>2</td><td rowspan="2">2,3</td><td rowspan="2">K1</td></tr>
<tr><td>A formula containing a part of the form $(x)P(x)or($ ∃ x)P(x), such a part is called an x-bound part of formula. Any variable appearing in an x bound part of the formula is called bound variable. Otherwise it is called free occurrence.</td><td>2</td></tr>
<tr><td>10</td><td>Some cats are black but all buffaloes are black</td><td>2</td><td>2,3</td><td>K2</td></tr>
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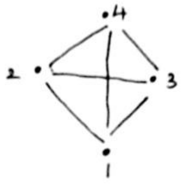
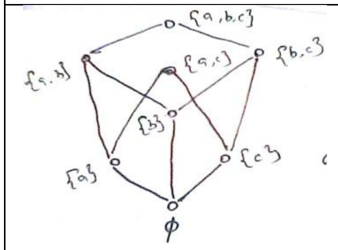
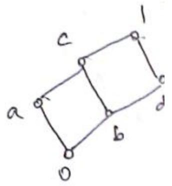
<tr><td></td><td>Let c(x): x is a cat
B(x): x is a buffaloes
b(x): x is black
$\exists x\,(c(x) \rightarrow b(x)) \wedge \forall x\,(B(x) \longrightarrow b(x)))$</td><td>2</td><td></td><td></td></tr>
<tr><td rowspan="2">11</td><td>Every student in this school is either good at studies or good in sports</td><td>2</td><td rowspan="2">2,3</td><td rowspan="2">K2</td></tr>
<tr><td>s(x): x is a student of this school
p(x): x is good at studies
q(x):x is good at sports
$\forall x(s(x) \longrightarrow (p(x) \vee q(x)))$</td><td>2</td></tr>
<tr><td rowspan="2">12</td><td>Use quantifiers to say that the square of every real number is non-negative</td><td>2</td><td rowspan="2">2,3</td><td rowspan="2">K2</td></tr>
<tr><td>$(\forall x)R\ (x^2 \geq 0)$</td><td>2</td></tr>
<tr><td rowspan="2">13</td><td>Symbolise:For every x, there exists y such that $x^2+y^2 \geq 100$</td><td>2</td><td rowspan="2">2,3</td><td rowspan="2">K2</td></tr>
<tr><td>$(x)(\exists y)(x^2+y^2) \geq 100$</td><td>2</td></tr>
<tr><td rowspan="2">14</td><td>All integers are rational numbers. Some integers are power of 2.some rational numbers are power of 2.</td><td>2</td><td rowspan="2">2,3</td><td rowspan="2">K2</td></tr>
<tr><td>I(x):x is an interger, R(x):x is a rational number, P(x): x is the power of 2
$\forall x\ I(x) \longrightarrow R(x)$, $\exists x\ I(x) \wedge P(x)$, $\exists x\ R(x) \wedge P(x)$</td><td>2</td></tr>
<tr><td rowspan="2">15</td><td>Show that $(\forall x)(H(x) \longrightarrow M(x)) \wedge H(S) \Longrightarrow M(S)$</td><td>2</td><td rowspan="2">2,3</td><td rowspan="2">K2</td></tr>
<tr><td><table>
<tr><th>S.No</th><th>Proposition</th><th>Explanation</th></tr>
<tr><td>1</td><td>$(\forall x)(H(x) \longrightarrow M(x))$</td><td>Rule P</td></tr>
<tr><td>2</td><td>$H(S)\ ) \Longrightarrow M(S)$</td><td>US Rule T</td></tr>
<tr><td>3</td><td>$H(S)$</td><td>Rule P</td></tr>
<tr><td>4</td><td>$M(S)$</td><td>$P, P \rightarrow Q \Rightarrow Q$, Modus ponens Rule T</td></tr>
</table></td><td>2</td></tr>
<tr><td colspan="2">UNIT-III LATTICES</td><td colspan="3"></td></tr>
<tr><td rowspan="2">1</td><td>Write a proof of Idempotent law</td><td>2</td><td rowspan="2">4</td><td rowspan="2">K1</td></tr>
<tr><td>Let aЄL for any x,y,z Є L
$x * y \leq z$ (1)
and if z≤x and z≤y then $z \leq x * y$ (2)
Now, take x=y=z=a
Then from (1) and (2) $a * a \leq a \text{ and } a \leq a * a$
$\therefore a = a * a$
$Similarly\ a * a = a$</td><td>1
1</td></tr>
</table>

2	State Hasse Diagram	2	4	K1
	Each vertex of A must be related to itself, so that the arrows from a vertex to itself are not necessary	1/2		
	If vertex b appears above vertex a and if vertex a is connected to vertex b by an edge then a r b, so direction arrows are not necessary	1/2		
	If vertex c is above verted a and if c is connected to a by a sequence of edges then a r c	½		
	The vertices (or nodes) are denoted by points rather than by circles	½		
3	Let A = {1,2,3,4} and let r be the relation ≤ on A. Draw the Hasse diagram of r.	2	4	K2
	r={{1,1}{1,2}{1,3}{1,4}{2,2}{2,3}{2,4}{3,3}{3,4}{4,4}}			
		2		
4	Let A={a,b,c} and P(A) be its power set. Draw a Hasse diagram of (P(A), ⊆)	2	4	K2
	P(A)={{a},{b},{c},{a,b},{a,c},{b,c},{a,b,c},∅}			
		2		
5	Write distributive lattice	2	4	K1
	Let (L,∨,∧) be a lattive under ≤. Then (L,∨,∧) is called distributive lattice ⇔ a∨(b∧c)=(a∨b)∧(a∨c); a∧(b∨c)=(a∧b)∨(a∧c)	1 1		
6	When is a lattice said to be bounded?	2	4	K2
	Let <L,∧,∨> be a given Lattice. It it has both '0' element and '1' element then I is said to be bounded Lattice. It is denoted by <L,∧,∨,0,1>	2		
7	Define a Boolean Algebra	2	4	K1

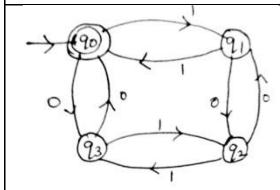
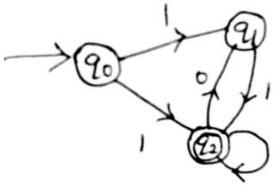
	A complemented distributive lattice is called Boolean Algebra. i.e., A Boolean algebra is distributive lattice with '0' element and '1' element in which every element has a complement.	2		
8	In a Lattice $<L,\leq>$ prove that $a\wedge(a\vee b)=a$, for all $a,b\epsilon L$	2	4	K2
	Since $a\wedge b$ is the glb of $\{a,b\}$, we have $a\wedge b\leq a$ (1) Obviously $a\leq a$ (2) From (1) and (2), we have $a\vee(a\wedge b)\leq a$ (3)	1		
	By defn of lub, we have $a\leq a\vee(a\wedge b)$ (4) from (3) and (4), $a\vee(a\wedge b)=a$, similarly $a\wedge(a\vee b)=a$	1		
9	Is there a Boolean Algebra with five elements? Justify your answer.	2	4	K2
	No, there is no Boolean Algebra wih five elements	1		
	Stone's representation theorem state that any Boolean Algebra is isomorphic to power set Algebra P(S). Therefore, the element is Boolean Algebra should be of the form 2^n	1		
10	Show that least upper bound of a subset B in a poset $< A, \leq >$ is unique of it exists.	2	4	K2
	Let $A=\{a_1, a_2\}$ Let u_1, u_2 be two least upper bounds of A. We have,(i) $a_1\leq u_1$ and $a_2\leq u_1$ [u_1 is upper bound (ii) $a_1\leq u_2$ and $a_2\leq u_2$ [u_2 is upper bound]	1		
	From (i) and (ii) u_1 and u_2 are the LUB $u_1\geq u_2$ & $u_2\geq u_1$ This implies that $u_1 = u_2$. Therefore, $< A, \leq >$ is unique	1		
11	Give an example of distributive lattice but not complemented	2	4	K2
	No complement exist o,b,c,d,1 The element 'a' is a complement of d and vice versa Therefore the above graph is not complemented  	2		
12	Define Lattice homomorphism	2	4	K1
	Let $<L_1,\wedge,\vee>$ and $<L_2, *,\oplus>$ be two given Lattices	1		

	A mapping f: $L_1 \rightarrow L_2$ is called Lattice homomorphism if, ∀ a,bϵ L			
	1. f(a) = f ∧b (a) ∗ f(b) 2. f(a∨b)= $f(a) \oplus f(b)$	1		
13	Prove that the Boolean identity ab+ab'=a is true	2	4	K2
	L.H.S = ab + ab'=a(b+b') = a.1=a=R.H.S	2		
14	Show that in any Boolean algebra (a+b)(a'+c)=ac+a'b+bc	2	4	K2
	Let (B,+,.,') be a Boolean algebra & a,b,cϵB L.H.S=(a+b)(a'+c) = (a+b)a'+(a+b)c = aa'+ba'+ac+bc = 0+ba'+ac+bc=a'b+ac+bc = R.H.S	2		
15	Every distributive lattice is modular.	2	4	K2
	Let (L,≤) be a distributive lattice For all a,b,cϵ L we have a∨(b∧c) = (a∨b)∧(a∨c)	1		
	Thus if a≤c then a∨c=c and a∨(b∧c)=(a∨b)∧ So if a≤c, the modular equation is satisfied and L is modular	1		
UNIT-IV LINEAR ALGEBRA				
1	Define Vector Space	2	5	K1
	A non-empty set V is said to be a vector space over a field F if V is an abelian group under an operation called addition which we denote by +	1		
	For every α ϵ F and v ϵ V, there is defined an element αv in V subject to the following conditions α(u+v) = αu+ αv for all u,v ϵ Vand α ϵ F (α+β)u=αu+βu for all u ϵ V and α, β ϵ F	½		
	α(βu)=(αβ)u for all u ϵ V and α, β ϵ F 1u=u for all u ϵ V	½		
2	Let V denote the set of all solutions of the differential equation $\mathbf{2\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 3y = 0}$. Then V is a vector space over R.	2	5	K2
	Let f,g ϵ V and α ϵ R. Then $2\frac{d^2f}{dx^2} - 7\frac{df}{dx} + 3f = 0 \; and \; 2\frac{d^2g}{dx^2} - 7\frac{dg}{dx} + 3g = 0$ $2\left[\frac{d^2f}{dx^2} + \frac{d^2g}{dx^2}\right] - 7\left[\frac{df}{dx} + \frac{dg}{dx}\right] + 3(f+g) = 0$	1		

	$2\frac{d^2}{dx^2}(f+g) - 7\frac{d}{dx}(f+g) + 3(f+g) = 0 \ \ Hence\ f+g \in V$ $Also\ 2\frac{d^2}{dx^2}(\alpha f) - 7\frac{d}{dx}(\alpha f) + 3\alpha f = 0\ Hence\ \alpha f \in V$ Since the operations are usual addition and scalar multiplication, the axioms of vector space are true. Hence V is a vector space over R.	1		
3	Prove that the intersection of two subspaces of a vector space is a subspace	2	5	K2
	Let A and B be two subspaces of a vector space V over a field F. Clearly 0 ϵ A∩ B and hence A∩ B is non-empty Now, let u,v ϵ A∩ B and α,β ϵF	1		
	Then u, v ϵ A and u,v ϵ B Therefore, αu+βv ϵ A and αu+βv ϵ B αu+βv ϵ A∩B. Hence A∩B is a subspace of V.	1		
4	Prove that the union of two subspaces of a vector space need not be a subspace	2	5	K2
	Let A = {(a,0,0)/a ϵ R}; B = {(0,b,0)/ b ϵ R} Clearly A and B are subspaces of R^3 However A∪B is not a subspace of R^3	1		
	For (1,0,0) and (0,1,0) ϵ A ∪B But (1,0,0) + (0,1,0)=(1,1,0) $\notin A \cup B$	1		
5	Define Linear Span	2	5	K1
	Let S be a non-empty subset of a vector space V. Then the set of all linear combinations of finite sets of elements of S is called the linear span of S and is denoted by L(S).	2		
6	Write the definition of finite dimensional		5	K2
	Let V be a vector space over a field F. V is said to be finite dimensional if there exists a finite subset S of V such that L(S)=V	2		
7	Any subset of a linearly independent set is linearly independent	2	5	K2
	Let V be a vector space over a field F Let S = $\{v_1,v_2,\ldots,v_n\}$be a linearly independent set Let S' be a subset of S. We take S' = $\{v_1,v_2,\ldots,v_k\}$ where k≤n Suppose S' is linearly dependent set.	1		
	Then there exist α_1, α_2, …α_k in F not all zero, such that $\alpha_1 v_1$, $\alpha_2 v_2$, …α_k v_k $+0v_{k+1}$ +….$+0v_n$=0 is non-trivial linear combination given the zero vector.	1		

	Here S is a linearly dependent set which is a contradiction. Hence S' is linearly independent			
8	Any set containing a linearly dependent set is also linearly dependent	2	5	K2
	Let V be a vector space over a field F. Let S be a linearly dependent set. Let $S' \supset S$.	1		
	If S' is linearly independent S is also linearly independent which is a contradiction Hence S' is linearly dependent.	1		
9	S={(1,0,0),(0,1,0),(1,1,1)} is a basis for $V_3(R)$	2	5	K2
	Let (a,b,c) = α(1,0,0) + β(0,1,0) + γ(1,1,1) Then α + γ = α, β + γ = b, γ = c Hence α= a-c and β = b-c	1		
	Thus (a,b,c)=(a-c)(1,0,0)+(b-c)(0,1,0)+c(1,1,1) Therefore, S is a basis for $V_3(R)$	1		
10	Define dimension	2	5	K1
	Let V be a finite dimensional vector space over a field F. The number of elements in any basis of V is called the dimension of V and is denoted by dimV.	2		
11	Define maximal linearly independent set.	2	5	K1
	Let V be a vector space and S = $\{v_1,v_2,\ldots,v_n\}$ be a set of independent vectors in v. Then S is called a maximal linearly independent set if for every v ϵ V-S, the set $\{v_1,v_2,\ldots,v_n\}$ is linearly dependent.	2		
12	Define minimal generating set	2	5	K1
	Let S = $\{v_1,v_2,\ldots,v_n\}$ be a set of vectors in V and let L(S)=V. Then S is called a minimal generating set if for any v_i ϵ S, $L(S-\{v_i\}) \neq V$	2		
13	Define Rank and Nullity	2	5	K1
	Let T:V→W be a linearly transformation. Then the dimension of T(V) is called the rank of T	1		
	The dimension of kerT is called the nullity of T.	1		
14	Let T:V→W be a linearly transformation. Then dimV= rankT + nullity T	2	5	K2
	W.K.T V/kerT=T(V) ∴ dimV-dim(KerT)=dim(T(V))	1		
	∴dimV-nullityT = rankT ∴dimV = nullity T + rank T	1		
15	Obtain the matrix representing the linear transformation T: $V_3(R) \rightarrow V_3(R)$ given by T(a,b,c)=(3a, a-b, 2a+b+c) w.r.t the standard basis $\{e_1,e_2,e_3\}$	2	5	K2
	$T(e_1) = T(1,0,0)=(3,1,2)=3e_1+e_2+e_3$ $T(e_2) = T(0,1,0)=(0,-1,1)= - e_2+e_3$	1		

<table>
<tr><td></td><td>$T(e_3) = T(0,0,1)=(0,0,1) = e_3$</td><td></td><td></td><td></td></tr>
<tr><td></td><td>The matrix T is $\begin{pmatrix}3 & 1 & 2\\0 & -1 & 1\\0 & 0 & 1\end{pmatrix}$</td><td>1</td><td></td><td></td></tr>
<tr><td colspan="5">UNIT-V FINITE STATE MACHINE AUTOMATA AND GRAMMARS</td></tr>
<tr><td rowspan="2">1</td><td>Definite finite automaton</td><td>2</td><td rowspan="2">6</td><td rowspan="2">K1</td></tr>
<tr><td>A finite automation is a 5-tuple M = {Q, ∑, δ, q_0, F} where
Q is a non empty finite set(states)
∑ is a finite non empty set whose elements are called input symbols
q_0 is the initial state
F the set of final states
δ:Q X ∑ →Q is called the next state function</td><td>2</td></tr>
<tr><td rowspan="2">2</td><td>What are the steps involved in the construction of state diagram</td><td>2</td><td rowspan="2">6</td><td rowspan="2">K2</td></tr>
<tr><td>represent the states of Q as nodes
initial state q_0 has an arrow pointing towards it
final states are indicated by double circles
there is a directed edge from node representing q to node representing q’ if δ(q,a) = q’ it is given the label</td><td>2</td></tr>
<tr><td rowspan="2">3</td><td>When is a string is accepted by a finite automaton</td><td>2</td><td rowspan="2">6</td><td rowspan="2">K2</td></tr>
<tr><td>The string S in ∑ is accepted by M if δ(q_0,X) = q for some q in F i.e., the string S is accepted by M if M reaches a final state on processing X.</td><td>2</td></tr>
<tr><td rowspan="2">4</td><td>When do you say that the languages is accepted or rejected by FSA</td><td>2</td><td rowspan="2">6</td><td rowspan="2">K2</td></tr>
<tr><td>When the symbols of an input string are fed into an FSM M sequentially they change the states of the automation successively and the automation ends up in a certain state. If the last state is an accepting state of an automation the string is said to be accepted or recognized by M otherwise it is rejected by M</td><td>2</td></tr>
<tr><td rowspan="2">5</td><td>When are two finite state automata are equivalent</td><td>2</td><td rowspan="2">6</td><td rowspan="2">K2</td></tr>
<tr><td>Two finite state are said to be equivalent if they accept the same language</td><td>2</td></tr>
<tr><td rowspan="2">6</td><td>Differentiate NFA and DFA</td><td>2</td><td rowspan="2">6</td><td rowspan="2">K2</td></tr>
<tr><td><table><tr><td>NFA</td><td>DFA</td></tr><tr><td>The transition function assigns a unique next state to every pair of state and input</td><td>The transition function assigns several next states to every pair of state and input</td></tr></table></td><td></td></tr>
<tr><td>7</td><td>Define non-deteministic finite automaton</td><td>2</td><td>6</td><td>K1</td></tr>
</table>

	A non-deterministic finite automation is a 5-tuple $M = \{Q, \sum, \delta, q_0, F\}$ where Q is a non empty finite set(states) $\sum$ is a finite non empty set whose elements are called input symbols q_0 is the initial state F the set of final states δ is the next state function $Q \times \sum \rightarrow 2^Q$			
8	Construct an FA accepting all strings over {0,1} having even number of 0's and even number of 1's	2	6	K2
		2		
9	Construct an NFA accepting all strings over {0,1} which end in one but does not contain the substring 00	2	6	K2
		2		
10	Definite context free grammar	2	6	K1
	A grammar G is said to a context free grammar or Type-2 grammar if each production $a \rightarrow b$ in G satisfy the condition $a \in V_N$ and $\lvert a\rvert \le \lvert b\rvert$	2		
11	Find L(G) where G has the productions $S \rightarrow aA$, $S \rightarrow bS$, $S \rightarrow a$, $S \rightarrow b$, $A \rightarrow bA$, $A \rightarrow bS$, $A \rightarrow b$	2	6	K2
	$S \Rightarrow aA \Rightarrow abA \Rightarrow abbS \Rightarrow abba$ $S \Rightarrow aA \Rightarrow abS \Rightarrow abb$; $S \Rightarrow aA \Rightarrow abA \Rightarrow abb$ L(G) = {abba, abb}	2		
12	Definite generation tree of a grammar	2	6	K1
	The generation tree (also called the parse tree) for a grammar $G = \{V_N, V_T, P, S\}$ is a tree such that	1		

	Every vertex has a label, which is an element of $V_N \cup V_T$, where T includes the null string λ also. The label of the root is S If a vertex is interior and has label A, then A ϵ V_N			
	If a vertex has label A and has n children with labels $X_1,X_2,\ldots X_n$ respectively from left to right then A → $X_1,X_2,\ldots X_n$ must be a production in P If a vertex has label λ then it is a leaf and the only son of its parent	1		
13	Define Top-down parsing, Bottom-up parsing	2	6	K1
	Generation of a string by beginning with start symbol and by successively applying the productions is called top-down parsing, the reverse of top-down parsing is called bottom-up parsing	2		
14	Define Ambiguity of a grammar	2	6	K1
	If in a grammar G a word W∈L(G) has more than one left most or right most derivation then the grammar G is said to be ambiguous	2		
15	Examine whether the following grammar G is ambiguous or not G={N,T,S,P}, where N = {S,A}, T = {a,b}, P consist of the rules S →aAb, S→abSb, S→a, A→bS, A→aAAb	2	6	K2
	S ⇒ aAb ⇒abSb⇒abab ; S⇒abSb⇒abab The word abab is generated by two left most derivation. Hence G is ambiguous	2		

PART – B (12 Mark Questions with Key)

S.No	Questions	Mark	COs	BTL
UNIT I – MATRIX THEORY				
1	Find the rank of the matrix $\begin{pmatrix} 3 & 1 & 4 & 6 \\ 2 & 1 & 2 & 4 \\ 4 & 2 & 5 & 8 \\ 1 & 2 & 2 & 2 \end{pmatrix}$	12	1	K3
	$R_4 \Leftrightarrow R_1$ $A \sim \begin{bmatrix} 1 & 2 & 2 & 2 \\ 2 & 1 & 2 & 4 \\ 4 & 2 & 5 & 8 \\ 3 & 1 & 4 & 6 \end{bmatrix}$ $R_2{:}R_2 - 2R_1;\ \ R_3{:}R_3 - 4R_1\ ;\ R_4{:}R_4 - 3R_1$	4		

	$\sim \begin{bmatrix} 1 & 2 & 2 & 2 \\ 0 & -3 & -2 & 0 \\ 0 & -6 & -3 & 0 \\ 0 & -5 & -2 & 0 \end{bmatrix}$ $R_3: R_3 - 2R_2\ ;\ R_4: 3R_4 - 5R_2$	3		
	$\sim \begin{bmatrix} 1 & 2 & 2 & 2 \\ 0 & -3 & -2 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 4 & 0 \end{bmatrix}$ $R_4: R_4 - 4R_3$ $\sim \begin{bmatrix} 1 & 2 & 2 & 2 \\ 0 & -3 & -2 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$	3		
	Rank of A = No. of non-zero rown in the last equivalent matrix = 3	2		
2	Solve (if possible) the equations $\boldsymbol{x + 2y - z = 3, 3x - y + 2z = 1, 2x - 2y + 3z = 2, x - y + z = -1}$	12	1	K3
	$[A, B] \sim \begin{bmatrix} 1 & 2 & -1 & 3 \\ 3 & -1 & 2 & 1 \\ 2 & -2 & 3 & 2 \\ 1 & -1 & 1 & -1 \end{bmatrix}$ $R_2: R_2 - 3R_1;\ \ R_3: R_3 - 2R_1\ ;\ R_4: R_4 - R_1$	3		
	$\sim \begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & -7 & 5 & -8 \\ 0 & -6 & 5 & -4 \\ 0 & -3 & 2 & -4 \end{bmatrix}$ $R_3: 7R_3 - 6R_2\ ;\ R_4: 7R_4 - 3R_2$	2		
	$\sim \begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & -7 & 5 & -8 \\ 0 & 0 & 5 & 20 \\ 0 & 0 & -1 & -4 \end{bmatrix}$ $R_4: 5R_4 + R_1$ $\sim \begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & -7 & 5 & -8 \\ 0 & 0 & 5 & 20 \\ 0 & 0 & 0 & 0 \end{bmatrix}$	2		
	$\therefore A \sim \begin{bmatrix} 1 & 2 & -1 \\ 0 & -7 & 5 \\ 0 & 0 & 5 \end{bmatrix}$ Rank of [A,B] = Rank of A = No. of unknowns=3 $\therefore \mathit{The\ system\ is\ consistent, and\ a\ unique\ solution}$	3		
	$x + 2y - z = 3$ $-7y + 5z = -8$ $5z = 20 \Leftrightarrow z = 4$ $-7\mathrm{y} + 5(4)\ = -8 \Leftrightarrow y = 4$ $x + 2(4) - 4 = 3\ \Leftrightarrow x =\ -1$ $\therefore \mathit{The\ solution\ is}\ x =\ -1, y = 4\ , z = 4$	2		

3	**Show that the matrix** $\begin{bmatrix}\mathbf{1} & \mathbf{3} & \mathbf{7}\\ \mathbf{4} & \mathbf{2} & \mathbf{3}\\ \mathbf{1} & \mathbf{2} & \mathbf{1}\end{bmatrix}$ $\boldsymbol{satisfies\ cayley-hamilton\ theorem}$	12	1	K3
	Characteristic equation is $\lambda^3 - 4\lambda^2 - 20\lambda - 35 = 0$	4		
	By theorem, $A^3-4A^2-20A-35I=0$	2		
	Now, $A^3-4A^2-20A-35I = \begin{pmatrix}135 & 152 & 232\\ 140 & 163 & 208\\ 60 & 76 & 111\end{pmatrix} - 4\begin{pmatrix}20 & 23 & 23\\ 15 & 22 & 37\\ 10 & 9 & 14\end{pmatrix} - 20\begin{pmatrix}1 & 3 & 7\\ 4 & 2 & 3\\ 1 & 2 & 1\end{pmatrix} - 35\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}$ $= \begin{pmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}$ Hence cayley hamilton theorem is verified	6		
4	Use cayley-Hamilton theorem to find the value of the matrix given by $A^8-5A^7+7A^6-3A^5+A^4-5A^3+8A^2-2A+I$, if the matrix $A = \begin{pmatrix}\mathbf{2} & \mathbf{1} & \mathbf{1}\\ \mathbf{0} & \mathbf{1} & \mathbf{0}\\ \mathbf{1} & \mathbf{1} & \mathbf{2}\end{pmatrix}$	12	1	K3
	The characteristic equation is $\lambda^3 - 5\lambda^2 + 7\lambda - 3 = 0$	4		
	By C-H theorem $A^3-5A^2+7A-3I=0$	1		
	Divide the g iven equation with characteristic equation we get, $\Phi(A) = (A^3-5A^2+7A-3I)(A^5+A) + A^2+A+I = A^2+A+I$	4		
	Now, $A^2 = \begin{pmatrix}5 & 4 & 4\\ 0 & 1 & 0\\ 4 & 4 & 5\end{pmatrix} \therefore \emptyset(A) = \begin{pmatrix}8 & 5 & 5\\ 0 & 3 & 0\\ 5 & 5 & 8\end{pmatrix}$	3		
5	Find the eigen values and eigen vectors of the matrix $\boldsymbol{A} = \begin{pmatrix}\mathbf{-2} & \mathbf{2} & \mathbf{-3}\\ \mathbf{2} & \mathbf{1} & \mathbf{-6}\\ \mathbf{-1} & \mathbf{-2} & \mathbf{0}\end{pmatrix}$	12	1	K3
	The characteristic equation of A is $\lambda^3 + \lambda^2 - 21\lambda - 45 = 0$	3		
	Eigen values are $\lambda = -3,-3,5$	3		
	Eigen vectors are $\begin{bmatrix}0\\ 3\\ 2\end{bmatrix}, \begin{bmatrix}3\\ 0\\ 1\end{bmatrix}, \begin{bmatrix}1\\ 2\\ -1\end{bmatrix}$	6		
6	Find the eigen values and eigen vectors of the matrix $\boldsymbol{A} = \begin{pmatrix}\mathbf{0} & \mathbf{1} & \mathbf{0}\\ \mathbf{1} & \mathbf{0} & \mathbf{1}\\ \mathbf{1} & \mathbf{1} & \mathbf{0}\end{pmatrix}$	12	1	K3
	The characteristic equation of A is $\lambda^3 - 3\lambda - 2 = 0$	3		

<table>
<tr><td></td><td>Eigen values are λ = -1,-1,2</td><td>3</td><td rowspan="2"></td><td rowspan="2"></td></tr>
<tr><td></td><td>Eigen vectors are $\begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}0\\1\\-1\end{bmatrix}, \begin{bmatrix}2\\-1\\-1\end{bmatrix}$</td><td>6</td></tr>
<tr><td colspan="5">UNIT II – LOGIC</td></tr>
<tr><td>1</td><td>Check whether ⌉ (P∨(Q∧R)) ⇔ ((P∨Q)∧(P∨R))is tautology or contradiction</td><td>12</td><td rowspan="4">2,3</td><td rowspan="4">K3</td></tr>
<tr><td></td><td>Consider ⌉ (P∨(Q∧R)) ⇔ ((P∨Q)∧(P∨R))=S</td><td></td></tr>
<tr><td></td><td><table>
<tr><th>P</th><th>Q</th><th>R</th><th>Q∧R</th><th>P∨(Q∧R)</th><th>⌉ (P∨(Q∧R))</th><th>P∨Q</th><th>P∨R</th><th>(P∨Q)∧(P∨R)</th><th>S</th></tr>
<tr><td>T</td><td>T</td><td>T</td><td>T</td><td>T</td><td>F</td><td>T</td><td>T</td><td>T</td><td>F</td></tr>
<tr><td>T</td><td>T</td><td>F</td><td>F</td><td>T</td><td>F</td><td>T</td><td>T</td><td>T</td><td>F</td></tr>
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</table></td><td>12</td></tr>
<tr><td></td><td>Hence it is contradiction</td><td></td></tr>
<tr><td>2</td><td>Find the PDNF for (P∧Q)∨(⌉ P∧R) ∨(Q∧R)</td><td>12</td><td rowspan="4">2,3</td><td rowspan="4">K3</td></tr>
<tr><td></td><td>Consider (P∧Q)∨(⌉ P∧R) ∨(Q∧R) = S</td><td></td></tr>
<tr><td></td><td><table>
<tr><th>P</th><th>Q</th><th>R</th><th>⌉ P</th><th>P∧Q</th><th>⌉ P ∧R</th><th>Q∧R</th><th>S</th><th>Minterm</th></tr>
<tr><td>T</td><td>T</td><td>T</td><td>F</td><td>T</td><td>F</td><td>T</td><td>T</td><td>P∧Q∧R</td></tr>
<tr><td>T</td><td>T</td><td>F</td><td>F</td><td>T</td><td>F</td><td>F</td><td>T</td><td>P∧Q∧⌉ R</td></tr>
<tr><td>T</td><td>F</td><td>T</td><td>F</td><td>F</td><td>F</td><td>F</td><td>F</td><td></td></tr>
<tr><td>T</td><td>F</td><td>F</td><td>F</td><td>F</td><td>F</td><td>F</td><td>F</td><td></td></tr>
<tr><td>F</td><td>T</td><td>T</td><td>T</td><td>F</td><td>T</td><td>T</td><td>T</td><td>⌉ P∧Q∧R</td></tr>
<tr><td>F</td><td>T</td><td>F</td><td>T</td><td>F</td><td>F</td><td>F</td><td>F</td><td></td></tr>
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<tr><td>F</td><td>F</td><td>F</td><td>T</td><td>F</td><td>F</td><td>F</td><td>F</td><td></td></tr>
</table></td><td>8</td></tr>
<tr><td></td><td>The PDNF of (P∧Q)∨(⌉ P∧R) ∨(Q∧R) is (P∧Q∧R)∨(P∧Q∧⌉ R)∨(⌉ P∧Q∧R)∨(⌉ P∧⌉ Q∧R)</td><td>4</td></tr>
<tr><td>3</td><td>Show that the premise E⟶S, S⟶H, A⟶⌉ H, E∧A are inconsistent</td><td>12</td><td rowspan="2">2,3</td><td rowspan="2">K3</td></tr>
<tr><td></td><td><table>
<tr><th>S.No</th><th>Proposition</th><th>Explanation</th></tr>
<tr><td>1</td><td>E∧A</td><td>Rule P</td></tr>
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<tr><td></td><td><table>
<tr><td>2</td><td>E</td><td>P∧Q⇒P, Rule T</td></tr>
<tr><td>3</td><td>E⟶S</td><td>Rule P</td></tr>
<tr><td>4</td><td>S</td><td>P, P⟶Q ⇒ Q, Modus Ponens</td></tr>
<tr><td>5</td><td>S⟶H</td><td>Rule P</td></tr>
<tr><td>6</td><td>H</td><td>P, P⟶Q ⇒ Q, Modus Ponens</td></tr>
<tr><td>7</td><td>A⟶⌉ H</td><td>Rule P</td></tr>
<tr><td>8</td><td>⌉ A</td><td>⌉ Q, P ⟶Q⇒ ⌉ P Modus tollens</td></tr>
<tr><td>9</td><td>A</td><td>P∧Q⇒Q [1], Rule T</td></tr>
<tr><td>10</td><td>A∧⌉ A</td><td>P,Q ⇒ P ∧ Q [8,9], Rule T</td></tr>
</table>
Hence it is inconsistent</td><td>6
6</td><td></td><td></td></tr>
<tr><td>4</td><td>Determine the validity of the following argument “My father praises me only it I can be proud of myself. Either I do well in sports or I cannot be proud of myself. If I study well, then I can’t do well in sports. Therefore if father praises me then I do not study well.
Let A:My father praises me; B: I can proud of myself; C: I do well in sports; D: I study well
Premise: A⟶B; C∨⌉ B; D⟶⌉ C; Conclusion:A⟶⌉ D</td><td>12</td><td rowspan="2">2,3</td><td rowspan="2">K3</td></tr>
<tr><td></td><td><table>
<tr><td>S.No</td><td>Proposition</td><td>Explanation</td></tr>
<tr><td>1</td><td>A</td><td>Rule P (assumed)</td></tr>
<tr><td>2</td><td>A⟶B</td><td>Rule P</td></tr>
<tr><td>3</td><td>B</td><td>P, P⟶Q ⇒ Q, Modus Ponens</td></tr>
<tr><td>4</td><td>C∨⌉ B</td><td>Rule P</td></tr>
<tr><td>5</td><td>C</td><td>Rule T, P, Q∨⌉ P⟹Q[Disjunctive syllogism]</td></tr>
<tr><td>6</td><td>D⟶⌉ C</td><td>Rule P</td></tr>
<tr><td>7</td><td>⌉ D</td><td>⌉ Q, P ⟶Q⇒ ⌉ P Modus tollens</td></tr>
<tr><td>9</td><td>A⟶⌉ D</td><td>CP</td></tr>
</table></td><td>6
6</td></tr>
<tr><td>5</td><td>Show that (x)(P(x)∨Q(x))⟹(x)P(x)∨ (∃x)Q(x)</td><td>12</td><td rowspan="2">2,3</td><td rowspan="2">K3</td></tr>
<tr><td></td><td>We shall use the indirect method of proof by assuming ⌉ ((x)P(x)∨ (∃x)Q(x)) as an additional premise</td><td></td></tr>
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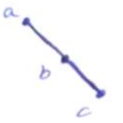
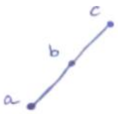
<table>
<tr><td>S.No</td><td>Proposition</td><td>Explanation</td></tr>
<tr><td>1</td><td>⌉ [(x)P(x)V ∃(x)Q(x)]</td><td>Negation Rule, Rule T</td></tr>
<tr><td>2</td><td>⌉ (x)P(x)∧⌉ ∃(x) Q(x)</td><td>Rule T, Demorgan’s law</td></tr>
<tr><td>3</td><td>⌉ (x)P(x)</td><td>Rule T, P∧Q⟹P</td></tr>
<tr><td>4</td><td>∃(x) ⌉ P(x)</td><td>Rule T,⌉ (x)P(x) ⇔ ∃(x)⌉ P(x)</td></tr>
<tr><td>5</td><td>⌉ P(y)</td><td>ES, Rule T</td></tr>
<tr><td>6</td><td>⌉ ∃(x)Q(x)</td><td>Rule T,(2),P∧Q⟹Q</td></tr>
<tr><td>7</td><td>(x)⌉ Q(x)</td><td>Rule T, ⌉ ∃(x)A(x) ⇔
(x)⌉ A(x)</td></tr>
<tr><td>8</td><td>⌉ Q(y)</td><td>US, Rule T</td></tr>
<tr><td>9</td><td>⌉ P(y)∧⌉ Q(y)</td><td>Rule T, P,Q⟹ P∧Q</td></tr>
<tr><td>10</td><td>⌉ [P(y)VQ(y)]</td><td>Rule T, Demorgan’s law</td></tr>
<tr><td>11</td><td>⌉ (x)[P(x)VQ(x)]</td><td>UG</td></tr>
<tr><td>12</td><td>(x)[P(x)VQ(x)]</td><td>Rule P</td></tr>
<tr><td>13</td><td>⌉ (x)[P(x)VQ(x)] ∧
(x)[P(x)VQ(x)]</td><td>Rule T,P,Q⟹ P∧Q, A∧⌉ A ⇔F</td></tr>
</table>
Hence it is a contradiction
</td>
<td>6
6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Show that from (∃x)(F(x)∧S(x))⟶(∀y)(M(y) ⟶W(y)), (∃y)(M(y) ∧⌉ W(y)) the conclusion (∀x)(F(x)⟶⌉ S(x)) follows</td>
<td>12</td>
<td>2,3</td>
<td>K3</td>
</tr>
</table>

S.No	Proposition	Explanation
1	(∃y)(M(y) ∧⌉ W(y))	Rule P
2	M(x)∧⌉ W(x)	ES, Rule T
	⌉ (⌉ M(x) V W(x))	Demorgan’s law, Rule T
3	⌉ (M(x)⟶W(x))	⌉ PVQ⇔ P ⟶Q, conditional, Rule T
4	(∃y) ⌉ (M(y) ⟶W(y))	EG, Rule T
5	(y)⌉ ((M(y) ⟶W(y))	∃(x) ⌉ A(x) ⟹ (x) ⌉ A(x), Rule T
6	(∃x)(F(x)∧S(x))⟶(∀y)(M(y)⟶W(y))	Rule P
7	⌉ ((∃x)(F(x)∧S(x))	Modus tollens
8	(x)⌉ (F(x)∧S(x))	∃(x) ⌉ A(x) ⟹ (x) ⌉ A(x), Rule T
9	⌉ (F(x)∧S(x))	US
10	F(x)⟶⌉ S(x)	Equivalence
11	(∀x)(F(x)⟶⌉ S(x))	UG

UNIT-III LATTICES

1	Let (L, ∨,∧) be a lattice and a,b,c,d ∈ ***L***. Prove that if a≤c & b≤d then (i) a∨b≤c∨d (ii) a∧b≤c∧d	12	4	K3
	Let c∨d is lub of c & d ⇒ c∨d is UB of c & d ⇒c≤c∨d & d≤c∨d	2		
	Given a≤c, c≤ c∨d ⇒ a≤ c∨d b≤d, d≤ c∨d⇒ b≤ c∨d	2		
	∴ c∨d is also an upperbound of a & b. Then a∨b ≤ c∨d	2		
	Let a∧b is glb of a & b ⇒ a∧b is LB of a & b a∧b ≤ a & a∧b ≤ b	2		
	Given a≤c, a∧b ≤ a ⇒ a∧b ≤ c b≤d, a∧b ≤ b ⇒ a∧b ≤ a	2		
	∴a∧b is also an lower bound of a & b Then a∧b ≤c∧d	2		
2	State and prove Isotonicity property in a Lattice	12	4	K3
	Statement: Let (L, ≤) be a lattice. For a,b,c∈ L b ≤ c ⇒ a∧b ≤ a∧c ; b ≤ c ⇒ a∨b ≤ a∨c	2		

	Case (i) To prove $a\wedge b \le a\wedge c$ $a\wedge b$ is the glb of a and b ⇒ $a\wedge b$ is the LB of a and b ⇒ $a\wedge b \le a$ & $a\wedge b \le b$ But $b\le c$ ⇒ $a\wedge b \le c$; $a\wedge b \le a$ ∴ $a\wedge b \le a\wedge c$	4		
	Case (ii) To prove $a\vee b \le a\vee c$ $a\vee c$ is the LUB of a and c ⇒ $a\vee c$ is the UB of a and c ⇒ $a\vee c \ge a$ & $a\vee c \ge c$ But $b\le c$ ⇒ $a\vee c \ge b$; $a\vee c \ge a$ ∴ $a\vee b \le a\vee c$	4		
3	State and prove Modular inequality in a lattice	12	4	K3
	Statement: Let $(L, \le)$ be a lattice. For $a,b,c \in L$ the inequality holds in L. $a\le c$ iff $a\vee(b\wedge c)\le(a\vee b)\wedge c$	2		
	Assume $a \le c$ W.K.T $b\wedge c \le c$ ∴ c is an UB of a & $b\wedge c$ $a\vee(b\wedge c) \le c$ (1) w.k.t $a \le a$, $b\wedge c \le b$ $a\vee(b\wedge c) \le a\vee b$ (2) From (1) & (2) $a\vee(b\wedge c) \le (a\vee b)\wedge c$ Hence proved.	4		
	Assume $a\vee(b\wedge c) \le (a\vee b)\wedge c$ $a \le a\vee(b\wedge c) \le (a\vee b)\wedge c \le c$	4		
4	Show that every chain is a distributive lattice	12	4	K3
	Let $(L, \le)$ be a chain. i.e., (i) $a \le b \le c$ (ii) $a \ge b \ge c$ ∀ $a,b,c \in L$ To prove $a\vee(b\wedge c) = (a\vee b)\wedge(a\vee c)$ $a\wedge(b\vee c) = (a\wedge b)\vee(a\wedge c)$	2		
	Case (i) $a \le b \le c$ LHS= $a\vee(b\wedge c) = a\vee b=b$ RHS=$(a\vee b)\wedge(a\vee c)= b\wedge c=b$ ⇒ $a\vee(b\wedge c) = (a\vee b)\wedge(a\vee c)$ Similarly $a\wedge(b\vee c) =$  $(a\wedge b)\vee(a\wedge c)$	4		
	Case (ii) $a \ge b \ge c$ LHS= $a\wedge(b\vee c) = a\wedge b=b$ RHS=$(a\wedge b)\vee(a\wedge c)= b\vee c=b$ ⇒ $a\wedge(b\vee c) = (a\wedge b)\vee(a\wedge c)$ Similarly $a\vee(b\wedge c) = (a\vee b)\wedge(a\vee c)$ 	4		

5	In a Boolean algebra B, prove the De Morgan's laws.	12	4	K3
	Let (L, $\oplus$, $\ast$) be Boolean lattice. (i.e.,) L is complemented and distributive lattice. The De-Morgan's laws are $\overline{a\oplus b} = \bar{a} \ast \bar{b}$; $\overline{a \ast b} = \bar{a}\oplus\bar{b}$, $\forall\ \bar{a}, a, b \in L$ Assume that $a, b \in L$, There exists $\bar{a}, \bar{b} \in L$ such that $a\oplus\bar{a} = 1; a \ast \bar{a} = 0; b\oplus\bar{b}; b \ast \bar{b} = 0$	2		
	Claim: $\overline{a\oplus b} = \bar{a} \ast \bar{b}$ Now $(a\oplus b)\oplus(\ \bar{a} \ast \bar{b}) = [\ (a\oplus b)\oplus\ \bar{a}\ ] * [(a\oplus b)\oplus\ \bar{b}\ ]$ $=[\ a \oplus \bar{a} \oplus b] \ast [a\oplus b\oplus\ \bar{b}] = [1\oplus b] \ast [a\oplus 1]$ $= 1 * 1 =1$ $(a\oplus b) \ast (\ \bar{a} \ast \bar{b}) = [\ (a\oplus b) \ast\ \bar{a}\ ] * [(a\oplus b) \ast\ \bar{b}\ ]$ $= [\ (a \ast \bar{a}\ )\ \oplus(b \ast \bar{a})] \ast [(a \ast \bar{b}) \oplus(b \ast \bar{b})]$ $= [\ 0 \oplus(b \ast \bar{a})] \ast [(a \ast \bar{b}) \oplus 0] = (b \ast \bar{a}) \ast (a \ast \bar{b})$ $=\ b \ast (\bar{a} \ast a) \ast \bar{b} =\ 0$ Hence claim (i) is proved.	5		
	Claim: $\overline{a \ast b} = \bar{a} \oplus \bar{b}$ Now $(a \ast b)\oplus(\ \bar{a}\oplus\bar{b}) = [\ (a \ast b)\oplus\ \bar{a}\ ] \oplus [(a \ast b)\oplus\ \bar{b}\ ]$ $=[\ a \oplus \bar{a}\ ) \ast (b\oplus\ \bar{a}\ )]\oplus[(a\oplus\bar{b}) \ast (b\oplus\bar{b})]$ $= [1\ast\ (b\oplus\ \bar{a}\ )]\oplus[(a\oplus\bar{b}) \ast 1] = (b\oplus\ \bar{a}\ )]\oplus[(a\oplus\bar{b})$ $= b\oplus(\bar{a}\oplus a)\oplus\bar{b}\ \ = = b \oplus\ \ \ \ 1 \oplus\bar{b}\ \ = \mathrm{b}\oplus\bar{b} = 1$ $(a \ast b) \ast (\ \bar{a}\oplus\bar{b}) = [\ (a \ast b) \ast\ \bar{a}\ ] \oplus [(a \ast b) \ast\ \bar{b}\ ]$ $= (\ a \ast \bar{a} \ast b) \oplus(a \ast b \ast \bar{b})$ $= (0 \ast b) \oplus(a \ast 0) = 0 \ast 0\ \ = 0$ Hence claim (ii) is proved. Hence the De-Morgan's laws are proved.	5		
6	Let (L, ≤) be a lattice in which $\ast$ and denotes the operations of meet and join respectively. For any a,b∈ ***L*** $\boldsymbol{a \leq b \Leftrightarrow a \ast b = a \Leftrightarrow a \oplus b = b}$	12	4	K3
	Let us assume that $a \leq b$ and also we know that $a \leq a$ $\therefore a \leq a \ast b \qquad (1)$ But, from definition of $a \ast b$, we have $a \ast b\ \ \leq a \qquad (2)$	4		

	Hence $a \leq b \Rightarrow a * b = a\ From$ (1)& (2)			
	Next, assume that $a * b = a$, but it is only possible if a ≤ b That is $a * b = a \Rightarrow a \leq b$ Compaining these two results, we get $a \leq b \Leftrightarrow a * b = a$	3		
	To show that $a \leq b \Leftrightarrow a \oplus b = b$ in a similar way. From $a * b = a, we\ have$ $b \oplus (a + b) = b \oplus a = a \oplus b$ $But\ b \oplus (a * b) = b$ $Hence\ a \oplus b = b\ follow\ that\ a * b = a$	2 3		
UNIT – IV LINEAR ALGEBRA				
1	R X R is a vector space over R under addition and scalar multiplication defined by $(x_1,x_2)+(y_1,y_2)=(x_1+y_1, x_2+y_2)$ and $\alpha(x_1,x_2) = (\alpha x_1,\alpha x_2)$	12	5	K3
	Clearly the binary operation + is commutative and associative ad (0,0) is the zero element The inverse of (x_1,x_2) is $(-x_1,-x_2)$ Hence (R X R, +) is an abelian group. Now, let u = (x_1,x_2) and v = (y_1,y_2) and let α, β ∈ R	2		
	Then $\alpha(u+v) = \alpha[(x_1,x_2) + (y_1,y_2)] = \alpha(x_1+ y_1, x_2+ y_2) = (\alpha x_1+ \alpha y_1, \alpha x_2+\alpha y_2) = \alpha(x_1,x_2) + \alpha(y_1,y_2) = \alpha u+\alpha v$	3		
	Now, $(\alpha+\beta)u=(\alpha+\beta) (x_1,x_2) = ((\alpha+\beta) x_1, (\alpha+\beta) x_2) = (\alpha x_1+ \beta x_1, \alpha x_2+ \beta x_2) = (\alpha x_1, \alpha x_2)+(\beta x_1, \beta x_2)$ $= \alpha(x_1,x_2) + \beta(x_1,x_2) = \alpha u + \beta u$	3		
	Also, $\alpha(\beta u) = \alpha(\beta(x_1,x_2)) = \alpha(\beta x_1, \beta x_2) = \alpha\beta x_1, \alpha\beta x_2 = (\alpha\beta)(x_1,x_2) = (\alpha\beta)u$ Obviously 1u = u Therefore R X R is a vector space over R.	3 1		
2	To prove that $V_3(R)$ the vectors (1,4,-2), (2,-1,3)and (-4,11,5) are linearly dependent	12		K3
	Let $\alpha_1(1,4,-2) + \alpha_2(-2,1,3) + \alpha_3(-4,11,5) = (0,0,0)$ $\therefore \alpha_1- 2\alpha_2 - 4\ \alpha_3 = 0$ (1) $4\alpha_1+\alpha_2 + 11\ \alpha_3 = 0$ (2) $-2\alpha_1+3\alpha_2 +5\alpha_3 = 0$ (3)	6	5	
	From (1) & (2), $\frac{\alpha_1}{-18} = \frac{\alpha_2}{-27} = \frac{\alpha_3}{9} = k\ (say)$ $\therefore \alpha_1= - 18k, \alpha_2 = - 27k , \alpha_3 = 9k$ These values of $\alpha_1, \alpha_2, \alpha_3$, for any k satisfy (3) also.	6		

	Taken k=1 we get $\alpha_1 = -18$, $\alpha_2 = -27$, $\alpha_3 = 9$ as a non-trivial solution. Hence the three vectors are linearly dependent.			
3	Let V be a vector space over F. A non-empty subset W of V is a subspace of V iff W is closed with respect to vector addition and scalar multiplication in V.	12	5	K3
	Let W be a subspace of V. Then W itself is a vector space and hence W is closed with respect to vector addition and scalar multiplication.	2		
	Conversely, let W be a non-empty subset of V such that $u,v \in W \Rightarrow u + v \in W$ And $u \in W$ and $\alpha \in F \Rightarrow \alpha u \in W$ We prove that W is a subspace of V	3		
	Since W is non-empty, there exists an element $u \in W$. $\therefore 0u = 0 \in W$ Also $v \in W \Rightarrow (-1)v = -v \in W$ Thus W contains 0 and the additive inverse of each of its element Hence W is an additive subgroup of V.	4		
	Also $u \in W$ and $\alpha \in F \Rightarrow \alpha u \in W$ Since the elements of W are the elements of V the other axioms of a vector space are true in W. Hence W is a subspace of V.	3		
4	Let V be a vector space over a field F. Let $S, T \subseteq V$. then $S \subseteq T \Rightarrow L(S) \subseteq L(T)$ $L(S \cup T) = L(S)+L(T)$ L(S) = S iff S is a subspace of V.	12	5	K3
	Let $S \subseteq T$. Let $s \in L(S)$ Then $s = \alpha_1 s_1+\alpha_2 s_2+\alpha_3 s_3+\ldots\ldots+\alpha_n s_n$ where $s_i \in S$ and $\alpha_i \in F$ Now, since $S \subseteq T$, $s_i \in T$ Hence $\alpha_1 s_1+\alpha_2 s_2+\alpha_3 s_3+\ldots\ldots+\alpha_n s_n \in L(T)$ Thus $L(S) \subseteq L(T)$	3		
	Let $s \in L(S \cup T)$ Then $s = \alpha_1 s_1+\alpha_2 s_2+\alpha_3 s_3+\ldots\ldots+\alpha_n s_n$ where $s_i \in S \cup T$ and $\alpha_i \in F$ Without loss of generality we can assume that $s_1, s_2, \ldots. s_m \in S$ and $s_{m+1}, \ldots., s_n \in T$ Hence $\alpha_1 s_1+\alpha_2 s_2+\alpha_3 s_3+\ldots\ldots+\alpha_m s_m \in L(S)$	3		

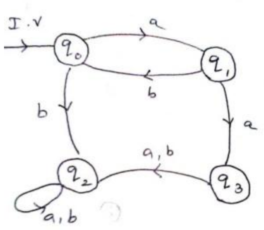
	And $\alpha_{m+1}s_{m+1}+\alpha_{m+2}s_{m+2}+\ldots+\alpha_n s_n \in L(T)$			
	$\therefore s=(\alpha_1 s_1+\alpha_2 s_2+\alpha_3 s_3+\ldots\ldots+\alpha_m s_m)+(\alpha_{m+1}s_{m+1}+\alpha_{m+2}s_{m+2}+\ldots+\alpha_n s_n) \in L(S)+L(T)$ Hence $L(S\cup T) \subseteq L(S)+L(T)$. Also by (a) $L(S) \subseteq L(S\cup T)$ and $L(T) \subseteq L(S\cup T)$ Hence $L(S)+L(T) \subseteq L(S\cup T)$ Hence $L(S\cup T) = L(S)+L(T)$	3		
	Let L(S)=S. W.K.T L(S)=S is a subspace of V. Conversely, let S be a subspace V. Then the smallest subspace containing S is S itself. Hence L(S)=S	3		
5	Check whether the following $V_3(R)$ vectors (1,2,1) , (2,1,0) and (1,-1,2) are linearly independent or not	6	5	K3
	Let $\alpha_1(1,2,1) + \alpha_2\,(2,1,0) + \alpha_3\,(1,-1,2) = (0,0,0)$ $(\alpha_1+2\,\alpha_2+\,\alpha_3,\ 2\,\alpha_1+\,\alpha_2-\,\alpha_3,\ \alpha_1+\,2\alpha_3) =(0,0,0)$	2		
	$\alpha_1+2\,\alpha_2+\,\alpha_3 = 0$ (1) $2\,\alpha_1+\,\alpha_2-\,\alpha_3 = 0$ (2) $\alpha_1+\,2\alpha_3 = 0$ (3)	2		
	Solving equations (1), (2) and (3) we get $\alpha_1 = \alpha_2 = \alpha_3 = 0$ The given vectors are linearly independent	2		
	Determine whether the following sets of vectors are linearly independent or not (1,1,0,0),(0,0,1,1),(1,0,0,4),(0,0,0,2) in $V_4(R)$	6		
	Let $\alpha_1(1,1,0,0) + \alpha_2\,(0,0,1,1) + \alpha_3\,(1,0,0,4) +\alpha_4\,(0,0,0,2) = (0,0,0,0)$ $(\alpha_1 + \alpha_3,\ \alpha_1,\ \alpha_2,\ \alpha_2 + 4\alpha_3 + 2\,\alpha_4) = (0,0,0,0)$	2		
	$\alpha_1 + \alpha_3 = 0$ (1) $\alpha_1 = 0$ (2) $\alpha_2 = 0$ (3) $\alpha_2 + 4\alpha_3 + 2\,\alpha_4 = 0$ (4)	2		
	Solving equations (1), (2),(3) and (4) we get $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 =0$ The given vectors are linearly independent	2		
6	$T:V_3(R) \rightarrow V_3(R)$ given by $T(a,b,c) = (3a+c, -2a+b, a+2b+4c)$w.r.t The standard basis The basis {(1,0,1),(-1,2,1), (2,1,1)} for both domain and range	12	5	K3
	$T(e_1) = T(1,0,0)=(3,-2,1)=3e_1-2e_2+e_3$ $T(e_2) = T(0,1,0)=(0,1,2)=\ e_2+2e_3$ $T(e_3) = T(0,0,1)=(1,0,4) = e_1+4e_3$	3		

	The matrix T is $\begin{pmatrix}3 & 2 & -1\\ 0 & 1 & 2\\ 1 & 0 & 4\end{pmatrix}$	3		
	T(1,0,1) = e_1+e_3 T(-1,2,1) = $-e_1+e_2+e_3$ T(2,1,1) = $2e_1+e_2+e_3$	3		
	The matrix T is $\begin{pmatrix}\frac{17}{4} & -\frac{3}{4} & \frac{1}{2}\\ \frac{35}{4} & \frac{15}{4} & -\frac{7}{2}\\ \frac{17}{2} & \frac{-3}{2} & 0\end{pmatrix}$	3		
UNIT-V FINITE STATE MACHINE AUTOMATA AND GRAMMARS				
1	Let M = ($\{q_0,q_1,q_2,q_3\}$, {a,b}, δ, q_0,$\{q_1\}$) where δ is given by $\delta(q_0,a) = q_1$, $\delta(q_0,b)=q_2$, $\delta(q_1,a) = q_3$, $\delta(q_1,b) = q_0$ $\delta (q_2,b)=q_2$, $\delta(q_3,a)=q_2$, $\delta(q_3,b)=q_2$, $\delta(q_2,a)=q_2$ (i) Represent M by its state table (ii) Represent M by its state diagram (iii) which of the following strings are accepted by M. ababa (2) aaaab (3) bbbaa	12	6	K3

To find state value

State	Input	
	a	b
q_0	q_1	q_2
q_1	q_3	q_0
q_2	q_2	q_2
q_3	q_2	q_2

Transition diagram



<table>
<tr><td></td><td>(1) $\delta(q_0, ababa) \Rightarrow \delta(q_1, baba)$
$\Rightarrow\delta(q_0, aba) \Rightarrow\delta(q_1, ba) \Rightarrow\delta(q_0, a) \Rightarrow\delta(q_1) \Rightarrow q_1 \in F \Rightarrow$Final state
The string ababa is accepted by M.</td><td></td><td></td><td></td></tr>
<tr><td></td><td>$\delta(q_0, aaaab) \Rightarrow \delta(q_1, aaab)$
$\Rightarrow\delta(q_3, aab) \Rightarrow\delta(q_2, ab) \Rightarrow\delta(q_2, b) \Rightarrow q_2 \notin F \Rightarrow$Final state
The string aaaab is not accepted by M.</td><td></td><td></td><td></td></tr>
<tr><td></td><td>$\delta(q_0, bbbaa) \Rightarrow \delta(q_2, bbaa)$
$\Rightarrow\delta(q_2, baa) \Rightarrow\delta(q_2, aa) \Rightarrow\delta(q_2, a) \Rightarrow q_2 \notin F \Rightarrow$Final state
The string bbbaa is not accepted by M.</td><td></td><td></td><td></td></tr>
<tr><td>2</td><td>Construct the state diagram for the automation M = $(\{q_0,q_1,q_2\}, \{a,b\}, \delta, q_0,\{q_2\})$ where δ is given by $\delta(q_0,a) = q_1$, $\delta(q_1,a)=q_1$, $\delta(q_2,a) = q_1$, $\delta(q_0,b) = q_2$
$\delta(q_1,b)=q_2$, $\delta(q_2,b)=q_0$ (i) Find its state table (ii) Find its state transition diagram (iii) find (a) $\delta(q_0,abab)$ (b) $\delta(q_2, bbab)$ (c) $\delta(q_1, \in)$ which string is accepted by M.</td><td>12</td><td>6</td><td>K3</td></tr>
<tr><td></td><td>State table
<table>
<tr><th rowspan="2">State</th><th colspan="2">Input</th></tr>
<tr><th>a</th><th>b</th></tr>
<tr><td>q_0</td><td>q_1</td><td>q_2</td></tr>
<tr><td>q_1</td><td>q_1</td><td>q_2</td></tr>
<tr><td>q_2</td><td>q_1</td><td>q_0</td></tr>
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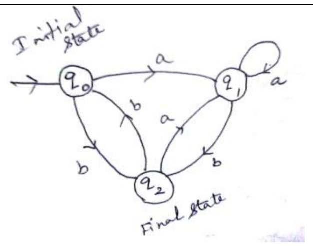
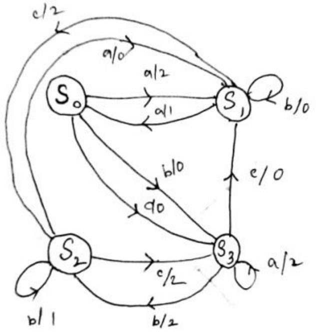
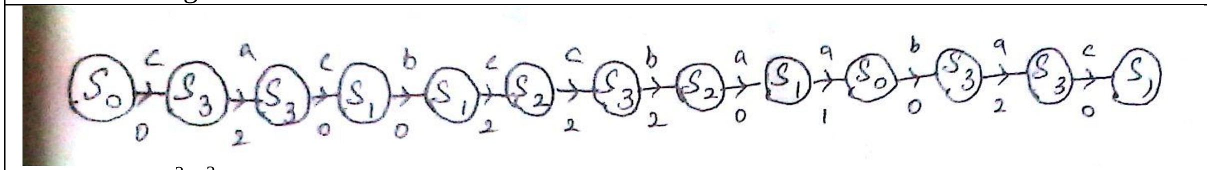
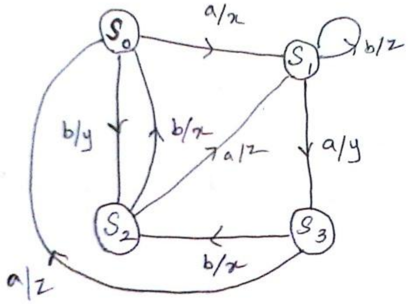
<tr><td></td><td>

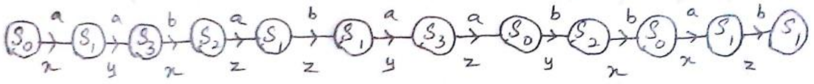
Transition diagram</td><td></td><td></td><td></td></tr>
<tr><td></td><td>(iii)(1) $\delta(q_0, abab) \Rightarrow \delta(q_1, bab)$
$\Rightarrow\delta(q_2, ab) \Rightarrow\delta(q_1, b) \Rightarrow q_2 \in F \Rightarrow$Final state
The string abab is accepted by M.</td><td></td><td></td><td></td></tr>
<tr><td></td><td>(2) $\delta(q_2, bbab) \Rightarrow \delta(q_{0,} bab)$
$\Rightarrow\delta(q_2, ab) \Rightarrow\delta(q_1, b) \Rightarrow q_2 \in F \Rightarrow$Final state</td><td></td><td></td><td></td></tr>
</table>

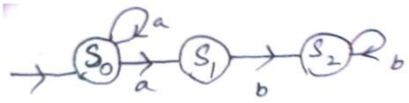
<table>
<tr><td rowspan="2"></td><td>The string bbab is accepted by M.</td><td></td><td></td><td></td></tr>
<tr><td>$\delta(q_1, \in) =q_1 \notin F \Rightarrow$Final state
The string $\in$ is not accepted by M.</td><td></td><td></td><td></td></tr>
<tr><td rowspan="4">3</td><td>Draw the state diagram of FSM with I = {a,b,c} , O={0,1,2} S = {s_0,s_1,s_2,s_3} and table is
<table>
<tr><td>I</td><td colspan="3">f</td><td colspan="3">G</td></tr>
<tr><td>S</td><td>a</td><td>b</td><td>c</td><td>a</td><td>b</td><td>C</td></tr>
<tr><td>S_0</td><td>S_1</td><td>S_3</td><td>S_3</td><td>2</td><td>0</td><td>0</td></tr>
<tr><td>S_1</td><td>S_0</td><td>S_1</td><td>S_2</td><td>1</td><td>0</td><td>2</td></tr>
<tr><td>S_2</td><td>S_1</td><td>S_2</td><td>S_3</td><td>0</td><td>1</td><td>2</td></tr>
<tr><td>S_3</td><td>S_3</td><td>S_2</td><td>S_1</td><td>2</td><td>2</td><td>0</td></tr>
</table>
Find the o/p cacbccbaabac</td><td>12</td><td rowspan="4">6</td><td rowspan="4">K3</td></tr>
<tr><td>Input State I = {a,b,c}
Output state O = {0,1,2}
State set S = {s_0,s_1,s_2,s_3}
Initial state M = S_0</td><td>2</td></tr>
<tr><td>

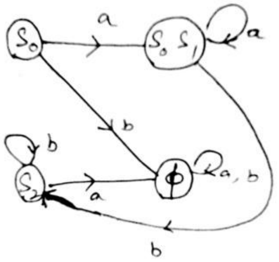
Transition diagram</td><td>5</td></tr>
<tr><td>

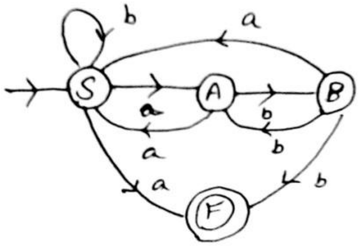
Output: $0\ 2\ 0^2\ 2^3\ 0\ 1\ 0\ 2\ 0$</td><td>5</td></tr>
<tr><td>4</td><td>The state table of a FSM M is given in the table:</td><td>12</td><td>6</td><td>K3</td></tr>
</table>

<table>
<tr><td></td><td>
<table>
<tr><td>f,g</td><td>a</td><td>b</td></tr>
<tr><td>S_0</td><td>S_1,x</td><td>S_2,y</td></tr>
<tr><td>S_1</td><td>S_3,y</td><td>S_1,z</td></tr>
<tr><td>S_2</td><td>S_1,z</td><td>S_0,x</td></tr>
<tr><td>S_3</td><td>S_0,z</td><td>S_2,x</td></tr>
</table>
Find Input set I, state set S, Output set O and initial state
Draw state diagram
Input O/P of word w=aababaabbab</td><td></td><td></td><td></td></tr>
<tr><td></td><td>Input set I = {a,b}
State set S = {s_0,s_1,s_2,s_3}
Output set O = {x,y,z}
Initial state = {s_0}</td><td>4</td><td></td><td></td></tr>
<tr><td></td><td>

Transition diagram</td><td>4</td><td></td><td></td></tr>
<tr><td></td><td>

Output = xyz z^2yzyx^2z</td><td>4</td><td></td><td></td></tr>
<tr><td>5</td><td>Construct the deterministic finite automation equivalent to NFA with state diagram

</td><td>12</td><td>6</td><td>K3</td></tr>
<tr><td></td><td>To find the state table for NFA
<table>
<tr><td>State</td><td>Input</td></tr>
</table></td><td>4</td><td></td><td></td></tr>
</table>

<table>
<tr><td rowspan="3"></td><td><table><tr><td></td><td>a</td><td>b</td></tr><tr><td>S_0</td><td>S_0, S_1</td><td>∅</td></tr><tr><td>S_1</td><td>∅</td><td>S_2</td></tr><tr><td>S_2</td><td>∅</td><td>S_2</td></tr></table></td><td></td><td rowspan="3"></td><td rowspan="3"></td></tr>
<tr><td>State table for DFA<table><tr><td rowspan="2">State</td><td colspan="2">Input</td></tr><tr><td>a</td><td>B</td></tr><tr><td>{S_0}</td><td>{S_0, S_1}</td><td>{∅}</td></tr><tr><td>{ S_0,S_1}</td><td>{S_0, S_1}</td><td>{S_2}</td></tr><tr><td>{S_2}</td><td>{∅}</td><td>{S_2}</td></tr><tr><td>∅</td><td>∅</td><td>∅</td></tr></table></td><td>4</td></tr>
<tr><td>Transition diagram for DFA
 </td><td>3</td></tr>
<tr><td rowspan="3">6</td><td>Find the DFA accepts the strings generated by the regular grammar G. P = {S →bs/aA/a; A→aS/bB;B→bA/aS/b} and S is the starting symbol</td><td></td><td rowspan="3">6</td><td rowspan="3">K3</td></tr>
<tr><td>Let the grammar L(G) = {V_N,V_T,S,P}
V_N = {S,A,B}
V_T={a,b}
S=starting</td><td></td></tr>
<tr><td>(i)Transition diagram for NFA</td><td></td></tr>
</table>



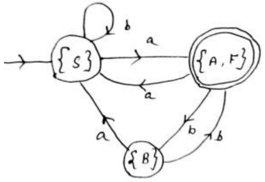
(ii)State table for NFA

State	Input	
	a	b
S	AF	S
A	S	B
B	S	A,F
F	-	-

(iii)State table for DFA

State	Input	
	a	B
{S}	{AF}	{S}
{A,F}	{S}	{B}
{B}	{S}	{A,F}

(iv)Transition diagram for DFA



PART – C (20 Mark Questions with Key)

S.No	Questions	Mark	COs	BTL
UNIT I – MATRIX THEORY				
1	Investigate for what values of a,b the simultaneous equations x+y+2z=2, 2x-y+3z=2, 5x-y+ax=b have	20	1	K4

	(i) no solution (ii) a unique solution (iii) an infinite number of solutions			
	$[A,B] \sim \begin{bmatrix}1&1&2&2\\2&-1&3&2\\5&-1&a&b\end{bmatrix} R_2{:}R_2-2R_1;\ \ R_3{:}R_3-5R_1$	3		
	$\sim \begin{bmatrix}1&1&2&2\\0&-3&-1&-2\\0&-6&a-10&b-10\end{bmatrix} R_3{:}R_3-2R_2$	3		
	$[A,B] \sim \begin{bmatrix}1&1&2&2\\0&-3&-1&-2\\0&0&a-8&b-6\end{bmatrix}$	3		
	$A \sim \begin{bmatrix}1&1&2\\0&-3&-1\\0&0&a-8\end{bmatrix}$ The last equivalent matrix is triangular	2		
	Case(i) $a\ \neq 8$ Rank of [A,B] = No. of non-zero rows=3 Rank of A = No. of non-zero rows=3; No. of unknowns = 3 ∴ The system is consistent and has a unique solution Case (ii) $a = 8$ & $b\ \neq 6$ Then Rank of [A,B] = 3, Rank of A =2 Rank of [A,B] ≠ Rank of A ∴ The system is inconsistent and has no solution	3		
	Case (iii) $a = 8$ & $b = 6$ Then Rank of [A,B] =2 , Rank of A =2, No. of unknowns=3 ∴ The system is consistent and has infinite number of solutions	3		
2	Find the characteristic equation of the matrix $\boldsymbol{A} = \begin{pmatrix}\mathbf{2}&\mathbf{-1}&\mathbf{2}\\\mathbf{-1}&\mathbf{2}&\mathbf{-1}\\\mathbf{1}&\mathbf{-1}&\mathbf{2}\end{pmatrix}$ $\boldsymbol{.Hence\ find\ A^{-1}\ and\ A^4}$	20	1	K3
	The characteristic equation is $\lambda^3 - 6\lambda^2 + 8\lambda - 3 = 0$	5		
	By C-H theorem, $A^3 - 6A^2 + 8A - 3I = 0$	2		
	Premultiplying by A^{-1} $we\ get$ $A^2 - 6A + 8I - 3A^{-1} = 0 \Leftrightarrow\ A^{-1} = \frac{1}{3}(A^2 - 6A + 8I)$	6		

	$$A^{-1} = \frac{1}{3}\begin{pmatrix} 3 & 0 & -3 \\ 1 & 2 & 0 \\ -1 & 1 & 3 \end{pmatrix}$$			
	Premultiplying by A on C-H theorem we get $A^4 = 6A^3 - 8A^2 + 3A = 6(6A^2 - 8A + 3I) - 8A^2 + 3A = 28A^2 - 45A + 18I$	4		
	$$A^4 = \begin{pmatrix} 124 & -123 & 162 \\ -95 & 96 & -123 \\ 95 & -95 & 124 \end{pmatrix}$$	3		
UNIT II –LOGIC				
1	With and without constructing the truth table obtain the product of sums canonical form of the formula (⌉ P⟶R)∧(Q↔P). Hence find the sum of products canonical form.	20	2,3	K4
	Let S⇔ (⌉ P⟶R)∧(Q↔P)			

Truth Table Method

P	Q	R	⌉ P	⌉ P⟶ R	Q↔ P	S	Minterm	Maxterm
T	T	T	F	T	T	T	P∧Q∧R	--
T	T	F	F	T	T	T	P∧Q∧⌉ R	--
T	F	T	F	T	F	F	--	⌉ PVQV⌉ R
T	F	F	F	T	F	F	--	⌉ PVQVR
F	T	T	T	T	F	F	--	PV⌉ QV⌉ R
F	T	F	T	F	F	F	--	PV⌉ QVR
F	F	T	T	T	T	T	⌉ P∧⌉ Q∧ R	--
F	F	F	T	F	T	F	--	PVQVR

	S⇔ (P∧Q∧R)V(P∧Q∧⌉ R)V(⌉ P∧⌉ Q∧R) (PDNF) S⇔(⌉ PVQV⌉ R)∧(PV⌉ QV⌉ R)∧(⌉ PVQVR)∧(PV⌉ QVR)∧(PVQVR) (PCNF)	5 5		
	Without Truth Table Method: Let S⇔ (⌉ P⟶R)∧(Q↔P) ⇔(PVR)∧(Q⟶P)∧(P⟶Q) ⇔(PVR)∧(⌉ QVP)∧(⌉ PVQ)			

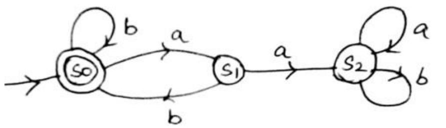
<table>
<tr><td></td><td>⇔(PVR)V(Q∧⌉ Q)∧(((⌉ QVP)V(⌉ R∧R))
⇔(PVRVQ)∧ (PVRV⌉ Q)∧ (⌉ QVPV⌉ R)∧ (⌉ QVPVR)∧ (⌉ PVRVQ)∧ (⌉ PVQV⌉ R)
⇔(PVRVQ)∧ (PVRV⌉ Q)∧ (⌉ QVPV⌉ R)∧ (⌉ PVRVQ)∧ (⌉ PVQV⌉ R) (PCNF)
⌉ S⇔(PVQ⌉ R)∧(⌉ PV⌉ QVR)∧ (⌉ PV⌉ QV⌉ R)
S⇔⌉ (⌉ S) ⇔⌉ [(PVQ⌉ R)∧(⌉ PV⌉ QVR)∧ (⌉ PV⌉ QV⌉ R)]
⇔(⌉ P∧⌉ Q∧R)V(P∧Q∧⌉ R)V(P∧Q∧R) (PDNF)</td><td>5
5</td><td></td><td></td></tr>
<tr><td>2</td><td>Show that P⟶(Q⟶P) ⇔⌉ P⟶(P⟶Q)</td><td>20</td><td rowspan="4">2,3</td><td rowspan="4">K1</td></tr>
<tr><td></td><td>
<table>
<tr><td>(i)</td><td>P⟶(Q⟶P)</td><td>Reasons</td></tr>
<tr><td></td><td>⇔P⟶(⌉ QV P)</td><td>Since Q⟶P ⇔⌉ QV P</td></tr>
<tr><td></td><td>⇔⌉ PV(⌉ QV P)</td><td>Since P⟶Q ⇔⌉ PV Q</td></tr>
<tr><td></td><td>⇔⌉ PV(PV⌉ Q)</td><td>Commutative</td></tr>
<tr><td></td><td>⇔ (⌉ PV P)V⌉ Q</td><td>Associative</td></tr>
<tr><td></td><td>⇔T V⌉ Q</td><td>Negation</td></tr>
<tr><td></td><td>⇔T</td><td>Since T V⌉ Q⇔ T</td></tr>
</table>
</td><td>10</td></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td></td><td>
<table>
<tr><td>(ii)</td><td>⌉ P⟶(P⟶Q)</td><td>Reasons</td></tr>
<tr><td></td><td>⇔⌉ P⟶(⌉ PV Q)</td><td>Since P⟶Q ⇔⌉ PV Q</td></tr>
<tr><td></td><td>⇔⌉ (⌉ P)V(⌉ PV Q)</td><td>Since P⟶Q ⇔⌉ PV Q</td></tr>
<tr><td></td><td>⇔PV(⌉ PV Q)</td><td>Double negation</td></tr>
<tr><td></td><td>⇔ (PV⌉ P) V Q</td><td>Associative</td></tr>
<tr><td></td><td>⇔T V Q</td><td>(PV⌉ P) ⇔T</td></tr>
<tr><td></td><td>⇔T</td><td>Since T V Q⇔ T</td></tr>
</table>
</td><td>10</td></tr>
</table>

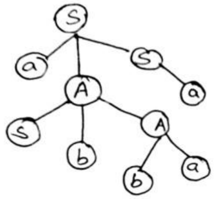
	From (i) ad (ii) we get P→(Q→P) ⇔ ⌉ P→(P→Q)			
UNIT-III LATTICES				
1	Prove that the direct product of two distributive lattice is a distributive lattice	20	4	K3
	Let (L, ⊕,*)& (S,∨,∧) be two distributive lattice w.r.t relations ≤ and ≤' respectively Then + & . in L X S is defined by: (a,b)+(c,d)=(a⊕c, b∨d) (a,b) . (c,d)=(a*c, b∧d) A relation α on L X S is defined by (x,y) α (z,u) ⇔ x≤z, y≤'u To prove that LXS is a distributive lattice	2		
	Case(i) LXS is a poset Let (a,b) ϵ L× S $\Rightarrow a \in L, b \in S$ $\Rightarrow a \le a, b \le' b$ $\therefore (a,b)\alpha(a,b)$ $\therefore \alpha$ *is reflexive in* $L \times S$	3		
	Let (a,b),(c,d) ϵ L× S such that $(a,b)\alpha(c,d) \& (c,d)\alpha(a,b)$ $\Rightarrow a \le c, b \le' d$ *and* $c \le a, d \le' b$ $\therefore$ *Now* $a \le c, c \le a \Rightarrow a = c$ $b \le' d, d \le' b \Rightarrow b = d$ $\therefore (a,b) = (c,d)$ *Hence* α *is anti symmetric.*	3		
	c) Let (a_1,b_1), (a_2,b_2), $(a_3,b_3) \in L \times S$ such that (a_1,b_1) α (a_2,b_2), (a_2,b_2) α (a_3,b_3) $\therefore a_1 \le a_2;\ b_1 \le 'b_2$ $a_2 \le a_3;\ b_2 \le 'b_3$ Now, $a_1 \le a_2, a_2 \le a_3 \Rightarrow a_1 \le a_3$ $b_1 \le' b_2, b_2 \le' b_3 \Rightarrow b_1 \le b_3$ $\therefore (a_1, a_3)\ \alpha\ (b_1, b_3)$ $\therefore\ \alpha$ *is transitive* Hence α *is a poset on* $L \times S$	3		
	Case (ii) To Prove $L \times S$ is a Lattice	3		

	$(a,b),(c,d) \in L \times S$ $\Rightarrow a,c \in L; b,d \in S$ Since L is a lattice,a & c have lub u_1 ϵ L, also a & c have glb l_1 ϵ L. b,d ϵ S; S is a lattice b&d have lub u_2 ϵ L b&d have glb l_2 ϵS Then (a,b)&(c,d) have lub (u_1,u_2) and (a,b) & (c,d) have glb (l_1,l_2) by def $\therefore L \times S$ is a Lattice			
	Case (iii) To Prove LXS is distributive lattice Let x= (a_1,b_1), y= (a_2,b_2), z = $(a_3,b_3) \in L \times S$ $\Rightarrow a_1,a_2,a_3 \in L \;\&\; b_1,b_2,b_3 \in S$ To Prove x.(y + z) = x.y + x.z $L.H.S = x.(y+z) = (a_1,b_1).[(a_2,b_2)+(a_3,b_3)]$ $=(a_1,b_1).[(a_2 \oplus a_3)+(b_2 \vee b_3)] = \left(a_1 * \left(a_2 \oplus a_3, b_1 \wedge (b_2 \vee b_3)\right)\right)$ $=\left((a_1 * a_2) \oplus (a_1 * a_3),(b_1 \wedge b_2) \vee (b_1 \wedge b_3)\right)$ $=(a_1 * a_2, b_1 \wedge b_2) + (a_1 * a_3, b_1 \wedge b_3)$ $=(a_1,b_1).(a_2,b_2)+(a_1,b_1).(a_3,b_3) = x.y + x.z = R.H.S$	3		
	To Prove x+(y . z) = (x+y) . (x + z) $L.H.S = x+(y.z) = (a_1,b_1)+[(a_2,b_2).(a_3,b_3)]$ $=(a_1,b_1)+[(a_2 * a_3),(b_2 \wedge b_3)] = \left(a_1 \oplus \left(a_2 * a_3, b_1 \vee (b_2 \wedge b_3)\right)\right)$ $=\left((a_1 \oplus a_2) * (a_1 \oplus a_3),(b_1 \vee b_2) \wedge (b_1 \vee b_3)\right)=(a_1 \oplus a_2, b_1 \vee b_2).(a_1 \oplus a_3, b_1 \vee b_3)$ $=(a_1,b_1)+(a_2,b_2).(a_1,b_1)+(a_3,b_3) = (x+y).(x+z) = R.H.S$	3		
2	State and Prove distributive inequality of Lattice	20	4	K1
	Statement: Let <L,∧,∨> be a given Lattice. For any a,b,c ϵ L the following inequality holds (1) a∨(b∧c)≤(a∨b)∧(a∨c) (2) a∧(b∨c)≥(a∧b)∨(a∧c)			
	Case(i) From defn of LUB a≤a∨b (1) and b∧c ≤ b ≤a∨b $\Rightarrow b \wedge c \leq a \vee b \qquad (2)$	2		
	From (1) and (2) a∨b is an UB of (a, b∧c) Hence a∨b ≥ a∨(b∧c) (A)	2		

	From defn of LUB $a\leq a\vee c$ (3) and $b\wedge c \leq c \leq a\vee c$ $\Rightarrow b \wedge c \leq a \vee c$ (4)	2		
	From (3) and (4) $a\vee c$ is an UB of (a, $b\wedge c$) Hence $a\vee c \geq a\vee(b\wedge c)$ (B)	2		
	From (A) and (B) $a\vee(b\wedge c)\leq(a\vee b)\wedge(a\vee c)$	2		
	Case(ii) $a \geq a\wedge b$ (1) and $b\vee c \geq b \geq a\wedge b$ $\Rightarrow b \vee c \leq a \wedge b$ (2)	2		
	From (1) and (2) $a\wedge b$ is an LB of (a, $b\vee c$) Hence $a\wedge b \leq a\wedge(b\vee c)$ (C)	2		
	$a\geq a\wedge c$ (3) and $b\vee c \geq c \geq a\wedge c$ $\Rightarrow b \vee c \leq a \wedge c$ (4)	2		
	From (3) and (4) $a\wedge c$ is an LB of (a, $b\vee c$) Hence $a\wedge c \leq a\wedge(b\vee c)$ (D)	2		
	From (C) and (D) $a\wedge(b\vee c)\geq(a\wedge b)\vee(a\wedge c)$	2		
UNIT-IV LINEAR ALGEBRA				
1	Let V be a vector space over F and W a subspace of V. Let V/W = {W+v/v ϵ V}. then V/W is a vector space over F under the following operations $(W+v_1) + (W+v_2) = W + v_1+v_2$ $A(W+v_1) = W + \alpha v_1$	20	5	K3
	Since W is a subspace of V it is a subgroup of (V, +) Since (V, +) is abelian, W is a normal subgroup of (V, +) so that (i) is well defined operation. Now, $W+ v_1 = W+ v_2 \Rightarrow v_1-v_2 \in W$ $\Rightarrow \alpha(v_1-v_2) \in W \Rightarrow \alpha v_1-\alpha v_2 \in W \Rightarrow \alpha v_1 \in W + \alpha v_2 \Rightarrow W+\alpha v_1 = W+\alpha v_2$ Hence (ii) is a well defined operation.	4		
	Now, let $W+ v_1$, $W+ v_2$, $W+ v_3 \in V/W$ Then $(W+ v_1)+[(W+ v_2)+(W+ v_3)] = (W+v_1) + (W+v_2+v_3) = W+v_1+v_2+v_3$ $= (W+v_1+v_2)+(W+v_3) = [(W+v_1) + (W+v_2)] + (W+v_3)$ Hence + is associative.	3		
	$W + 0 = W \in V/W$ is the additive identity element . For $(W+ v_1)+(W+0) = W+ v_1= (W+0) + (W+ v_1)$ for all $v_1 \in V$ Also $W-v_1$ is the additive inverse of $W + v_1$	3		

	Hence V/W is a group under +			
	Further $(W+v_1)+(W+v_2) = W + v_1+v_2 = W + v_2 + v_1 = (W+v_2) + (W+v_1)$ Hence V/W is an abelian group.	2		
	Now let α, β ϵ F $\alpha[(W+v_1)+(W+v_2)] = \alpha(W + v_1+v_2) = W + \alpha(v_1+v_2) = W+ \alpha v_1+ \alpha v_2 = (W+ \alpha v_1)+(W+ \alpha v_2) = \alpha (W+ v_1)+ \alpha (W+ v_2)$	3		
	$(\alpha+\beta) (W+v_1) = W + (\alpha+\beta)v_1 = W+ \alpha v_1 + \beta v_1 = (W+ \alpha v_1)+(W+ \beta v_1) = \alpha (W+ v_1) + \beta (W+ v_1)$	3		
	$\alpha[\beta(W+v_1)] = \alpha(W+ \beta v_1) = W + \alpha\beta v_1= (\alpha\beta) (W+v_1)$			
	$1(W+v_1) = W+1v_1 = W+v_1$ Hence V/W is a vector space. The vector space V/W is called the quotient space of V by W	2		
2	Let V be a finite-dimensional vector space over a field F. Let A and B be subspace of V Then dim(A+B) = dimA + dimB- dim (A∩B)	20	5	K3
	W.K.T A + B is a subspace of V containing A Hence $\frac{A+B}{A}$ is also a vector space over F An element of $\frac{A+B}{A}$ is of the form A + (a+b) where a ϵ A and b ϵ B. But A + a = A Hence an element of $\frac{A+B}{A}$ is of the form A + b Now, consider f : B $\rightarrow\frac{A+B}{A}$ defined by f(b) = A + b Clearly f is onto	6		
	Also $f(b_1+b_2) = A + (b_1+b_2) = (A + b_1) +(A + b_2) = f(b_1) + f(b_2)$ And $f(\alpha b_1) = A + \alpha b_1 = \alpha(A+b_1) = \alpha f(b_1)$ Hence f is a linear transformation	4		
	Let K be the kernel of f. Then K = {b/b ϵ B, A + b = A } Now, A + b = A iff b ϵ A Hence K = A∩B (w.k.t $\frac{V}{V_1} \cong W$) $\Rightarrow \frac{B}{A\cap B} \cong \frac{A+B}{A}$ $Hence \quad dim \left(\frac{A+B}{A}\right) = dim \left(\frac{B}{A\cap B}\right)$ $\dim(A+B) - dimA = dimB - \dim(A\cap B)$	10		

	$\dim(A+B) = dimA + dimB - \dim(A \cap B)$			
UNIT-V FINITE STATE MACHINE AUTOMATA AND GRAMMARS				
1	Explain deterministic and nondeterministic automation with example. How to convert an NFA to equivalent DFA Construct a finite state automation that accept all strings over {a,b} in which every a is followed by b	20	6	K1
	In a finite state automation that transition function assigns a unique next state to every pair of state and input then the FSA is called a deterministic finite state automation If the transition function assigns several next states to every pair of state and input then FSA is called Non-deterministic finite state automation(Any example)	8		
	Let the given NFA be M = {S, I, f, s_0, A} and let M' be the required equivalent DFA each state of M' will be a subset of S. accordingly the initial state of M' is {s_0}. The set of input symbols of M' is same as I. *If* $\{s_{i_1}, s_{i_2}, \dots s_{i_k}\}$ *is a state of* M' *and a is the input symbol fed into the* M' *is the union of the sets* $f(s_{i_1}), f(s_{i_2}), \dots f(s_{i_k})$ *where f is the ttransion* Thus, the states of M' are some or all the subsets of S including the empty set ϕ. The final states of M' are those states that contain the final states of M.	4		
	The simplest word ab should be acceptable by the FSA so the FSA should move from s_0 to s_1 when the input symbol is a. when the input symbol is b at s_1 the FSA should move to the accepting state which may be taken as s_0 itself. When a is input at s_1 the FSA should move to the nonfinal trap state s_2. When b is input at s_0 FSA should move to s_0 itself the input symbols a and b at s_2 should take FSA from s_2 to itself	4		
		4		
2	Write a notes on (i) Generation tree of a grammar with example (ii) Derivation tree for a string (iii) Types of derivation (iv) In the grammar with productions S →0B|1A; A→0|0S|1AA; B→1|1S|0BB for the sring 00110101, find (a) left most derivation (b) right most derivation and (c) derivation tree	20	6	K1
	Generation tree of a grammar The generation tree (also called the parse tree) for a grammar G = {V_N,V_T,P,S} is a tree such that Every vertex has a label, which is an element of $V_N \cup V_T$, where T includes the null string λ also. The label of the root is S If a vertex is interior and has label A, then A ϵ V_N If a vertex has label A and has n children with labels X_1,X_2,…X_n respectively from left to right then A →	4		

<table>
<tr><td></td><td>$X_1,X_2,\ldots X_n$ must be a production in P
If a vertex has label λ then it is a leaf and the only son of its parent</td><td></td><td></td><td></td></tr>
<tr><td></td><td>Example: Let G be a grammar with production rules S→aAS|a; A→SbA|ba
 </td><td>2</td><td></td><td></td></tr>
<tr><td></td><td>Derivation tree for a string
Derivation tree for a string S of a language L(G) is a rooted tree whose root is S(starting symbol in G) and whose leaves from left to right by concatenation constitute S(the given string)</td><td>2</td><td></td><td></td></tr>
<tr><td></td><td>Types of derivation
Left most/Rightmost derivation: Leftmost/Rightmost derivation of an expression (or string) in L(G) starts with the symbol S of G and using the production rules of G. If we always replace the left most/rightmost non terminal symbol by a production till the last, then the resulting expression is said to have been got by leftmost/rightmost derivation</td><td>4</td><td></td><td></td></tr>
<tr><td></td><td>
<table>
<tr><th colspan="2">Left most derivation</th><th colspan="2">Rightmost derivation</th></tr>
<tr><td>S →</td><td>0B</td><td>S →</td><td>0B</td></tr>
<tr><td>→</td><td>00BB</td><td>→</td><td>00BB</td></tr>
<tr><td>→</td><td>001SB</td><td>→</td><td>00B1S</td></tr>
<tr><td>→</td><td>0011AB</td><td>→</td><td>00B10B</td></tr>
<tr><td>→</td><td>00110SB</td><td>→</td><td>00B101S</td></tr>
<tr><td>→</td><td>001101AB</td><td>→</td><td>00B1010B</td></tr>
<tr><td>→</td><td>0011010B</td><td>→</td><td>00B10101</td></tr>
<tr><td>→</td><td>00110101</td><td>→</td><td>00110101</td></tr>
</table>
</td><td>4</td><td></td><td></td></tr>
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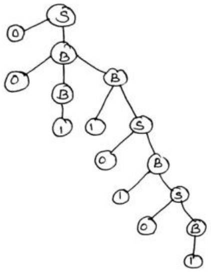
	Derivation tree for the string is 	4		